DISCRETE-TIME HIGHER ORDER SLIDING MODE:

THE CONCEPT AND THE CONTROL

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by

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CERTIFICATE

This is to certify that the thesis entitled **Discrete-time Higher Order Sliding Mode Control: The Concept and the Control** submitted by **Nalin Kumar Sharma** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Dr. S. Janardhanan

Department of Electrical Engineering,

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(Supervisor)

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Nalin Kumar Sharma

ABSTRACT

Higher order sliding mode has been successfully applied to many control problems due to its attractive features such as improved sliding accuracy and smoother control input. This thesis investigates the existence of higher order sliding mode in discrete-time systems and propounds a new concept of “Discrete-time Higher Order Sliding Mode”. A definition of “Discrete-time Higher Order Sliding Mode” is proposed and a control law is designed through the proposed concept for an uncertain linear-time invariant system. The behaviour of the closed-loop system is analysed analytically and experimentally validated on a rectilinear plant.

Next, this thesis proposes a technique for designing a sliding surface such that when confined to the surface, the closed-loop system has optimality in the linear quadratic sense. The corresponding optimal discrete-time higher order sliding mode control is designed for an LTI system with partial state information. Hence, the implementation of the proposed technique requires lesser sensors. Moreover, the uncertainties in the system may not always satisfy the matching condition. So, a discrete-time higher order sliding control is designed in the thesis considering the unmatched uncertainties in the system.

Further, this thesis also proposes an adaptive discrete-time higher order sliding mode control. The proposed adaptive technique does not require the knowledge of the bound on uncertainty in the system and requires less control effort. Thus, the practicability of discrete-time higher order sliding mode has been improved.

Many practical systems do not have a deterministic nature. For these type of stochastic systems, this thesis defines a discrete-time higher order sliding mode in probabilistic sense and named as “Stochastic Discrete-time Higher Order Sliding Mode”. A control input is designed for a stochastic system such that the proposed stochastic discrete-time sliding mode is achieved.

Keywords: Higher order sliding mode, Discrete-time systems, Adaptive discrete-time sliding mode, Optimal discrete-time sliding mode, Stochastic systems.

सार

बेहतर सटीकता और अविरल नियंत्रण इनपुट जैसी आकर्षक विशिष्टताओं के कारण उच्च नियंत्रण स्लाइडिंग मोड को कई नियंत्रण समस्याओं पर सफलतापूर्वक लागू किया गया। यह थीसिस डिस्क्रीट-टाइम में हायर आर्डर स्लाइडिंग मोड की सम्भवता की जांच करता है और "डिस्क्रीट-टाइम में हायर आर्डर स्लाइडिंग" की एक नई अवधारणा प्रदान करता है। 'डिस्क्रिट-टाइम हायर ऑर्डर स्लाइडिंग मोड' की परिभाषा को प्रस्तावित किया गया है और प्रस्तावित अवधारणा के माध्यम से नियंत्रण इनपुट डिज़ाइन किया गया है। क्लोज-लूप सिस्टम के व्यवहार का विश्लेषण किया गया है और प्रयोगात्मक रूप से इसकी पुष्टि एक रेक्टिलिनर संयंत्र पर की गयी है।

इसके बाद, यह थीसिस एक स्लाइडिंग सतह को डिजाइन करने के लिए एक तकनीक इस तरह प्रस्ताव करती है कि स्लाइडिंग मोड कि स्थिति में क्लोज-लूप सिस्टम लीनियर क्वाड्रटिक सेन्स में ऑप्टीमल हो। यह ऑप्टीमल हायर ऑर्डर स्लाइडिंग मोड नियंत्रण आंशिक जानकारी वाले एलटीआई सिस्टम के लिए डिज़ाइन किया गया है। इसलिए प्रस्तावित तकनीक के कार्यान्वयन के लिए कम सेंसर की आवश्यकता होती है। इसके अलावा, सिस्टम में अनिश्चितता हमेशा मैचिंग कंडीशन को पूरा नहीं कर सकती है इसलिए, सिस्टम में अनमैचड अनिश्चितता पर विचार करते हुए एक डिस्क्रिट-टाइम हायर ऑर्डर स्लाइडिंग मोड नियंत्रण इनपुट डिज़ाइन किया गया है।

इसके अलावा, यह थीसिस एक अडाप्टिव डिस्क्रिट-टाइम हायर ऑर्डर स्लाइडिंग मोड नियंत्रण का भी प्रस्ताव करती है। प्रस्तावित अडाप्टिव तकनीक को सिस्टम में अनिश्चितता के बाध्यता के ज्ञान की आवश्यकता नहीं होती है और कम नियंत्रण प्रयास की आवश्यकता होती है। इस प्रकार, डिस्क्रिट-टाइम हायर ऑर्डर स्लाइडिंग मोड की व्यावहारिकता में सुधार किया गया है।

कई व्यावहारिक प्रणालियों के मैथमेटिकल मॉडल में एक निर्धारिक प्रकृति नहीं होती है। इस प्रकार के स्टोकास्टिक सिस्टम के लिए, यह थीसिस प्रोबबिलिस्टिक सेन्स में डिस्क्रिट-टाइम हायर ऑर्डर स्लाइडिंग मोड को परिभाषित करता है और जिसे 'स्टोकास्टिक डिस्क्रिट-टाइम हायर ऑर्डर स्लाइडिंग मोड' के रूप में नामित किया गया है। नियंत्रण इनपुट एक स्टोकास्टिक सिस्टम के लिए डिज़ाइन किया गया है जिससे कि प्रस्तावित स्टोकास्टिक स्टोकास्टिक डिस्क्रिट-टाइम हायर ऑर्डर स्लाइडिंग मोड प्राप्त किया जा सके।

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Nomenclature

Acronyms

VSS Variable structure systems

ADHOSM Adaptive Discrete-time Higher Order Sliding Mode

DHOSM Discrete-time Higher Order Sliding Mode

DSM Discrete-time Sliding Mode

HOSM Higher Order Sliding Mode

LTI Linear Time-invariant

ODHOSM Optimal Discrete-time Higher Order Sliding Mode

SDHOSM Stochastic Discrete-time Higher Order Sliding Mode

SMC Sliding Mode Control

Symbols

$\bar{s}$ Higher order sliding function

ϵ Switching gain

$\mathbb{E}\{.\}$ Expectation operator

$\tilde{d}$ Disturbance vector in a system

ξ	The vector containing the sliding function and its higher order terms
A	State matrix in discrete-time model of LTI system
B	Input matrix in discrete-time model of LTI system
d	The effect of the disturbance vector on the sliding function
d^*	bound of d
d_0	The mean of disturbance bounds d_u and d_l
d_l	Lower bound of d
d_u	Upper bound of d
I	Identity matrix
k	Sampling instant
k_1	The variable denoting the gain
k_2	The variable denoting the gain
m	Number of the inputs in an LTI system
n	Number of the states in an LTI system
P	Probability measure
r	Sliding order
T	Sampling time
u	Control input.
V	Lyapunov function
V_w	Variance of signal w