

**LOW VELOCITY IMPACT ON GLASS FIBRE  
REINFORCED EPOXY COMPOSITES:  
EXPERIMENTS AND SIMULATION**

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**by**

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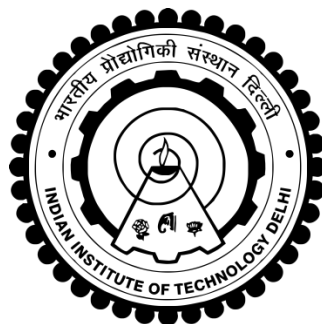
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**Submitted**

**in fulfilment of the requirements of the degree of**

**Doctor of Philosophy**

**to the**



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# CERTIFICATE

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This is to certify that the thesis entitled "**Low Velocity Impact on Glass Fiber Reinforced Epoxy Composites : Experiments and Simulation**" being submitted by **Mr. Kiran Kumar Namala** to the **Indian Institute of Technology Delhi** for the award of degree of Doctor of Philosophy is a record of bonafide research work carried out by him under our supervision and guidance. The thesis, in our opinion has reached the standard of fulfilling the requirements for the award of degree of **Doctor of Philosophy**.

The research report and the results presented in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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KIRANKUMAR NAMALA

IIT DELHI

## ABSTRACT

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Composite material when used for specific stiffness and specific strength applications are susceptible to damage when subjected to impact loads. To understand the response of these materials under impact loading experiments and numerical simulations are performed in this work.

For simulating the response of composite laminate numerically properties such as moduli, strength and failure energies are needed which have to be determined experimentally. The properties of unidirectional composite such as tensile, compressive, shear, intra-laminar fracture toughness and interlaminar fracture toughness (mode I and mode II) are determined according to ASTM standards. The values of these properties are determined and compared with those available in the literature for similar fiber volume fraction. Iosipescu shear test was performed for finding shear modulus and shear strength. Three point bend test was done for finding fracture energy ( $G_I$ ) in Mode I for matrix cracking and fiber failure (parallel and perpendicular to fiber orientation). The double cantilever beam (mode I) and the end notch flexure (mode II) tests were performed to determine interlaminar fracture toughness  $G_I$  and  $G_{II}$ . The determined properties were used in the numerical simulation of low-velocity impact laminates for unidirectional as well as cross-ply laminates. A continuum damage mechanics based constitutive law with provision for stiffness degradation of each lamina due to intra-laminar damage is used in the simulations. Intra-laminar damage initiation was modelled using Hashin's and Shurman-Puck failure criteria. Damage evolution for different intra-laminar failure modes are related to the strain. The constitutive material model was incorporated as a subroutine which was linked to a commercial Finite Element (FE) code.

Low Velocity Impact (LVI) experiments were performed on unidirectional laminate and cross-ply laminate of E-glass/epoxy composites. The energy, contact forces and displacement plots with respect to time were studied for the drop test. The damage observed as back face signature on the back face of both cross-ply and unidirectional laminate were plotted. The composite was speckled randomly and the impact phenomenon was recorded by using a high-speed camera. The DIC data on the surface of the plate was analyzed to obtain displacement and strains on the surface of the plate. The damage observed as back face signature on the back face of laminate, energy, contact forces and displacement plots with respect to time were determined and compared with FE results.

**Key Words:** Composites, Finite element simulation, Digital Image Correlation, damage, Low Velocity Impact.

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## NOMENCLATURE

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|                         |                                                                       |
|-------------------------|-----------------------------------------------------------------------|
| $\varepsilon_f$         | Strain at failure                                                     |
| $\alpha$                | Angle of V-notch                                                      |
| $\alpha$                | Approach of two elastic bodies in contact                             |
| ${}^n\bar{\sigma}_{yy}$ | Average in plane transverse stress in $n^{\text{th}}$ lamina          |
| ${}^n\bar{\sigma}_{xz}$ | Average transverse shear stress in XZ plane in $n^{\text{th}}$ lamina |
| ${}^n\bar{\sigma}_{yz}$ | Average transverse shear stress in YZ plane in $n^{\text{th}}$ lamina |
| $a$                     | Crack length                                                          |
| $a_0$                   | Initial crack length                                                  |
| $a_i$                   | Acceleration                                                          |
| $d$                     | Depth of crack in the specimen                                        |
| $d_f^c$                 | Damage variable for fiber compression                                 |
| $d_f^t$                 | Damage variable for fiber tension                                     |
| $d_m^c$                 | Damage variable for matrix compression                                |
| $d_m^t$                 | Damage variable for matrix tension                                    |
| $\mu_t^n, \mu_l^n$      | Friction coefficients                                                 |
| $\delta_m^{\max}$       | Maximum effective separation value during loading                     |
| $\delta_m^f$            | Effective separation at complete damage                               |
| $\delta_m^0$            | Effective separation at damage initiation                             |
| $\varepsilon_{11}^0$    | Strain at which damage initiates                                      |
| $\varepsilon_{11}^{ft}$ | Failure strain at fiber tension                                       |
| $\varepsilon_{11}^{fc}$ | Failure strain at fiber compression                                   |
| $\alpha$                | Angle of Rotation about optical axis                                  |
| $\alpha_0$              | Fracture angle                                                        |
| $\rho$                  | Density of composite body                                             |
| $\delta$                | Load point displacement                                               |
| $\delta$                | Deflection of the plate                                               |
| $\beta$                 | Stereo angle                                                          |
| $\gamma$                | Tilt angle                                                            |
| $\theta$                | Fracture angle                                                        |

|                                      |                                                             |
|--------------------------------------|-------------------------------------------------------------|
| $\delta_n, \delta_s, \delta_t$       | Separation values of normal, shear and traction loading     |
| $\delta_n^f, \delta_s^f, \delta_t^f$ | Final relative displacements in normal, shear and traction. |
| $\delta_n^0, \delta_s^0$             | Onset displacement at damage initiation                     |
| $\xi, \eta, \zeta$                   | Curvilinear coordinate system of an iso-parametric element  |
| $\nu_{12}$                           | Poisson's ratio                                             |
| $\rho_c$                             | Density of the composite                                    |
| $\alpha_{cr}$                        | Critical indentation depth                                  |
| $\rho_f$                             | Density of the fiber                                        |
| $\sigma_i$                           | Stresses in contracted notation                             |
| $\sigma_e$                           | Effective stress                                            |
| $\varepsilon_i$                      | Strains in contracted notation                              |
| $\nu_{ij}$                           | Poisson's ratio                                             |
| $\varepsilon_{ij}$                   | Strain tensor                                               |
| $\gamma_{ij}$                        | Strain tensor ( $i \neq j$ )                                |
| $\sigma_{ij}$                        | Stress tensor/ Stress component                             |
| $\tau_{ij}$                          | Stress tensor ( $i \neq j$ )                                |
| $\tau$                               | Interlaminar shear stress                                   |
| $\alpha_m$                           | Maximum indentation                                         |
| $\Delta$                             | Correction factor                                           |
| $\Delta t$                           | Size of time step                                           |
| $[K], \mathbf{K}$                    | Stiffness matrix                                            |
| $[M], \mathbf{M}$                    | Mass matrix                                                 |
| $b_i$                                | Body force per unit volume                                  |
| $b$                                  | Specimen width                                              |
| $b$                                  | Distance between the notch roots                            |
| $B$                                  | Thickness of specimen                                       |
| $C$                                  | Compliance                                                  |
| $C_0, C_1, C_2, C_3, C_4,$           | Compliance coefficients                                     |
| $c_0, c_1, c_2, c_3, c_4$            | Calibration coefficients                                    |
| $C_{ij}$                             | Stiffness                                                   |

|                        |                                                               |
|------------------------|---------------------------------------------------------------|
| $C_{ijkl}$             | Elasticity matrix in generalized Hooke's law for anisotropic  |
| $D$                    | Scalar damage variable                                        |
| $E_1, E_{11}$          | Young's Modulus of the composite material along fiber         |
| $E_2, E_{22}$          | Young's Modulus of the composite material along transverse    |
| $E_3, E_{33}$          | Young's Modulus of the composite material in out of plane     |
| $E_{ij}$               | Young's moduli                                                |
| $e_M, e_D$             | Matrix cracking and delamination failure criteria identifiers |
| $e_{xx}$               | Strain along fiber direction                                  |
| $F, F_c$               | Contact Force                                                 |
| fps                    | Frame per second                                              |
| $g$                    | Acceleration due to gravity                                   |
| $g$                    | Energy dissipated in softening per unit volume                |
| $G, G_f$               | Fracture energy per unit area of the composite                |
| $G_{1+}, G_{1-}$       | Fracture energy for longitudinal failure in tension and       |
| $G_{2+}, G_{2-}$       | Fracture energy for transverse failure in tension and         |
| $G_6$                  | Fracture energy for transverse failure in shear               |
| $G_c$                  | Critical fracture energy                                      |
| $G_I, G_{II}, G_{III}$ | Fracture energy in mode I, mode II, mode III                  |
| $G_{ij}$               | Shear moduli                                                  |
| $h$                    | Drop height                                                   |
| $h$                    | Width of specimen                                             |
| $k$                    | Stiffness of spring                                           |
| $K$                    | Stiffness matrix                                              |
| $K_n, K_s, K_t$        | Stiffness in normal, shear and traction directions            |
| $L$                    | Half of span length                                           |
| $l^*$                  | Characteristic length                                         |
| $m$                    | Meters                                                        |
| $m/s$                  | Metre/second                                                  |
| $m_1, m_2$             | Mass of colliding spheres                                     |
| $m_c$                  | Mass of composite                                             |

|                                                |                                                                 |
|------------------------------------------------|-----------------------------------------------------------------|
| $m_f$                                          | Mass of reinforcement                                           |
| $m_i, M, M_i, m$                               | Mass of impactor                                                |
| $M$                                            | Mass matrix                                                     |
| $m_m$                                          | Mass of matrix                                                  |
| mm                                             | Millimetres                                                     |
| N                                              | Newton                                                          |
| $N_i$                                          | Standard continuous shape functions                             |
| $n_c$                                          | Number of nodes in the element                                  |
| N-m                                            | Newton metre                                                    |
| P                                              | Applied Load                                                    |
| S                                              | Span of specimen length                                         |
| $S_{12}$                                       | In-plane shear strength                                         |
| $S_{11}^t$                                     | Longitudinal tensile strength                                   |
| $S_{11}^c$                                     | Transverse tensile strength                                     |
| $S_{22}^t$                                     | Transverse tensile strength                                     |
| $S_{23}^A$                                     | Shear strength in fracture plane                                |
| sec                                            | Second(s)                                                       |
| $S_i$                                          | Transverse shear strength ( i = interlaminar )                  |
| $S_{ij}$                                       | Compliances                                                     |
| t                                              | Time                                                            |
| t, h                                           | Thickness of specimen                                           |
| $T_{eff}$                                      | Effective traction stress at the damage initiation              |
| $t_n, t_s, t_t$                                | Stress generated due to normal, shear and traction loading with |
| $\bar{t}_n, \bar{t}_s, \bar{t}_t$              | Traction stress without damage                                  |
| $T_x, T_y, T_z$                                | Components of resultant base distance between the cameras       |
| $t_n^0, t_s^0$                                 | Interface normal tensile strength and shear strength            |
| U                                              | Strain energy                                                   |
| ${}^t u$ and ${}^{t+\Delta t} u$               | Displacement at time $t$ and $t + \Delta t$                     |
| ${}^t \dot{u}$ and ${}^{t+\Delta t} \dot{u}$   | Velocity at time $t$ and $t + \Delta t$                         |
| ${}^t \ddot{u}$ and ${}^{t+\Delta t} \ddot{u}$ | Acceleration at time $t$ and $t + \Delta t$                     |

|                 |                                                        |
|-----------------|--------------------------------------------------------|
| $\ddot{u}_I$    | Acceleration of the impactor                           |
| $v$             | Velocity                                               |
| $V_c$           | Volume of the composite                                |
| $V_f$           | Volume fraction of fiber                               |
| $V_m$           | Volume fraction of matrix                              |
| $w$             | Displacement of a point                                |
| $W$             | Width of the specimen                                  |
| $W$             | Length of specimen in shear test                       |
| $x$             | Compressed length of spring during impact              |
| $X_1, Y_1, Z_1$ | Coordinate system of image plane of camera 1           |
| $X_2, Y_2, Z_2$ | Coordinate system of image plane of camera 2           |
| $X, X_c$        | In plane longitudinal tensile and compressive strength |
| $Y, Y_c$        | In plane transverse tensile and compressive strength   |
| $Z, Z_c$        | Out of plane tensile and compressive strength          |
|                 |                                                        |
| AOI             | Area of Interest                                       |
| ASTM            | American Society for Testing and Materials             |
| CMOD            | Crack Mouth Opening Displacement                       |
| CMOS            | Conducting Metal Oxide Semiconductor                   |
| DCB             | Double Cantilever Beam                                 |
| DIC             | Digital Image Correlation                              |
| ENF             | End Notch Flexure                                      |
| LVI             | Low Velocity Impact                                    |