SOME STUDIES ON STATIC AND DYNAMIC RESPONSE OF POLAR ORTHOTROPIC, LAYERED, MODERATELY THICK CIRCULAR PLATES/AND SHALLOW SPHERICAL SHELLS

by

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jñeyaḥ sa nitya-sannāysi
yo na dveṣṭi na kāṅkṣati
nirdvandvo hi Mahā-bāho
sukhaṁ bandhāt pramucvate

Translation

One who neither hates nor desires the fruits of his activities is known to be always renounced. Such a person, free from all dualities, easily overcomes material bondage and is completely liberated, O Arjuna.

[5.3 Bhagavad Gita as it is]

Dedicated
to
My Father

Who has always worked for the happiness of human being

CERTIFICATE

This is to certify that the thesis entitled "**Some Studies on Static and Dynamic Response of Polar Orthotropic, Layered, Moderately Thick Circular Plates and Shallow Spherical Shells**" being submitted by **Mr. Sandeep Kumar** to the **Indian Institute of Technology, Delhi** for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a record of bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirement of the Doctor of Philosophy Degree.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

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09/03/97

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Sandeep Kr

(SANDEEP KUMAR)

ABSTRACT

With the advancement of the fibre production technique, the use of fibre reinforced composite materials in modern engineering structures such as missiles, aircrafts, submarines and automobiles has increased, considerably. This is due to its high directional strength-to-weight ratio, high stiffness-to-weight ratio, fatigue life, etc. To use anisotropic composite materials in the engineering structures efficiently, there needs the study of behaviour of plates and shells. The classical Love-Kirchhoff's hypothesis of plates and shells, which has been used extensively for thin isotropic plates and shells, neglects the effects of transverse shear. This theory of plates and shells does not adequately describes the behaviour of thick laminated structures. It breaks down in the situations where the ratio of Youngs modulus to shear modulus is high and even in the case of higher modes analysis. It underpredicts deflections and overpredicts natural frequencies and buckling loads for thick laminated plates and shells. In all these cases the effect of transverse shear is quite significant and theory which incorporate, it must be employed so that better estimation of deflections, frequencies and buckling loads can be provided.

The object of the thesis is to study the large amplitude static and dynamic response and buckling of polar orthotropic, layered, moderately thick circular plates and shallow spherical shells using first order shear deformation theory with parabolic distribution of transverse shear stress through the thickness. Among the various shear deformation theories, the first order theory seems to provide the best compromise

between accuracy and computational efficiency.

Considering the effects of transverse shear and rotatory inertia, the governing equations of motion for shells are derived and expressed in terms of normal deflection, slope and stress function. A methodology based on Chebyshev polynomial approximation to analyse the nonlinear boundary value problem in circular domain is developed. These nonlinear partial differential equations of motion are linearized using quadratic extrapolation technique. The inertia and dissipative terms are evaluated by employing either Houbolt or Newmark-β implicit time marching scheme. Convergence study has been carried out. In order to validate the present methodology of solution the results have been compared with the results available in the literature.

The axisymmetric moderately large amplitude static and dynamic response of polar orthotropic, symmetrically and antisymmetrically laminated, cross-ply, moderately thick circular plates are studied. The results, depicting the effects of transverse shear, rotatory inertia, base radius to thickness ratio, number of layers on circular plates subjected to uniformly distributed normal loadings are presented. Clamped immovable edge, simply supported immovable edge, clamped movable edge and simply supported movable edge conditions are considered. Three different materials namely glass-epoxy, boron-epoxy and graphite-epoxy whose properties are given in Appendix-I are considered in the present analysis. The dynamic responses of circular plates due to step and sinusoidal loadings are studied.

The study of axisymmetric large amplitude static and dynamic response of polar orthotropic, layered, cross-ply moderately thick shallow spherical shells

subjected to uniform normal loadings has been carried out. The effects of control space variables; material orthotropy, base radius to thickness ratio, shell rise to thickness ratio, shell rise to base radius ratio, number of layers, boundary conditions are presented. The effects of transverse shear and rotatory inertia are shown. The step and sinusoidal loadings are considered. Snap through buckling response have been studied and buckling loads are estimated.

Lastly, an attempt has been made to develop a methodology based on Chebyshev series solution technique for the solution of nonlinear boundary value problems in rectangular domain. The nonlinear static and dynamic response of isotropic rectangular plates and doubly curved shells have been studied.

The following papers have been prepared till now from the present thesis:

1. Y. Nath and K. Sandeep, "Postbuckling of symmetrically laminated moderately thick axisymmetric shallow spherical shells", Intl. J. Mech. Sci., vol. 35, n. 11, pp. 965-975, 1993.
2. Y. Nath and K. Sandeep, "The stability of the Houbolt time marching scheme in nonlinear system", J. Sound Vib., vol. 174, n. 4, pp. 565-569, 1994.
3. Y. Nath and K. Sandeep, "Large amplitude response of layered circular plates", ASCE, J. Engrg. Mech., vol. 121, n. 1, pp. 37-49, 1995.
4. Y. Nath and K. Sandeep, "Chebyshev series solution to nonlinear boundary value problems in rectangular domain", Computer Methods Appl. Mech. Engrg., vol. 125, pp. 41-52, 1995.

5. K. Sandeep and Y. Nath, "Chebyshev series solution to boundary value problem in structural mechanics", Proc. 15th CANCAM 95, Univ. Victoria, vol. 2, pp. 860-861, 1995.

6. K. Sandeep and Y. Nath, "Large amplitude static and dynamic response of unsymmetrically laminated circular plates and shallow spherical shells", Recent Advances in Solid Mechanics, Proc. CEASM 95, IIT-M, pp. 232-237, 1995.

7. Y. Nath and K. Sandeep, "Nonlinear analysis of unsymmetrically laminated moderately thick axisymmetric structures", Mech. Res. Commun. (Communicated).

8. Y. Nath and K. Sandeep, "Nonlinear dynamic response of axisymmetric thick laminated shallow spherical shells", Intl. Computer Struc. (Communicated).

9. Y. Nath and K. Sandeep, "Effect of transverse shear on static and dynamic buckling of antisymmetrically laminated polar orthotropic shallow spherical shells", Composite Struc. (Communicated).

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