

THREE DIMENSIONAL FINITE ELEMENT  
ANALYSIS OF BRANCHING TUNNELS

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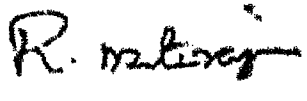
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CERTIFICATE

This is to certify that the thesis entitled 'Three Dimensional Finite Element Analysis of Branching Tunnels' being submitted by Mr. Pharis Var Tharaja to the Indian Institute of Technology, Delhi for the award of the degree of DOCTOR OF PHILOSOPHY, is a result of his research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the standard fulfilling the requirements for the award of the degree of Doctor of Philosophy. The research report and the results presented in the thesis have not been submitted, in part or in full, to any other University or Institute, for the award of any degree or diploma.

  
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## ABSTRACT

A number of hydro-electric projects are at planning, design and construction stages in India to exploit the huge untapped hydro-potential in the Himalayan region. An important component of these projects is the single or sometimes branching pressure tunnel. Although extensive technical backup is available to the designers for design of a single tunnel, there is paucity of literature on stress analysis and lining design for the junction zones associated with the branching of tunnels. The simplified design approach in vogue is to ignore the contribution of surrounding rock in sharing the loads from the liners. Further, in the absence of suitable analytical methods, the designers have a tendency to play safe and provide the heaviest possible reinforcement around the junction which besides being uneconomical has other undesirable consequences. The objective of the present work has been to make a rational analysis and suggest a design approach for tackling the problem of branching tunnels as it really exists.

In the present work, the circular tunnel branchings have been analysed using three-dimensional finite element method considering linear elastic, isotropic and homogeneous behaviour of the material of lining and surrounding rock mass. To suit the complicated geometry and curved boundaries, twenty noded isoparametric brick elements are used to discretise the concrete liner. While considering the joint action of concrete and steel liners for the case of uncracked concrete, the steel liner is taken to act as a membrane and is represented by eight noded isoparametric elements. However in the case when concrete is assumed to have been cracked, the steel liner is represented by twenty noded three dimensional elements.

The rock surrounding the liner is simulated by Winkler Model. Using the coefficient of subgrade reaction of rock, the passive reactive forces offered by the rock are evaluated as equivalent nodal stiffnesses and added to the nodes of the liner. The effect of cracked back-filled concrete is worked out as equivalent stiffness and is added to that with the rock.

A versatile computer program is developed for automatic generation of mesh with capability to analyse all cases associated with branching of tunnels for both with and without rock participation conditions, viz. 90 degrees and oblique branchings, bifurcation and trifurcation, thickened and normal junctions, concrete and steel liners acting independently and together and cracked back filled concrete in case of steel liner encased in rock. To evaluate boundary conditions for the main and branch tunnels in respect of all these alternatives, Lamé's thick cylinder theory has been extended and plane strain derivations for the case of a single circular liner taking into account the interaction of concrete liner and rock; steel liner and concrete liner; steel liner, cracked backfilled concrete and rock; and steel liner, uncracked backfilled concrete and rock have been carried out.

Realistic and practical dimensions of the main and branch tunnels and the values of deformation modulus of the rock surrounding the branchings, have been taken for analysis. The results of analyses are presented in a user friendly format for the tunnel designer. Keeping in view that all the tunnel designers may not have access to a sophisticated computer program, the results of analyses are also summed up to provide simple guidelines for the assessment of stress concentration zones. Further, empirical correlations are presented to predict the peak

hoop stresses for critical longitudinal planes with respect to both with and without rock participation cases.

The present work has also contributed to a recognition of the assistance of the surrounding rock, even of average rock modulus value, in relieving stress concentration effect around the junctions of the liner. The branching tunnels, when encased in rock of average quality, need not be excessively strengthened with heavy reinforcement as is the prevailing practice. It is more important to attend to construction specifications so that the rock and liner act together to withstand high internal hydrostatic pressure.

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