A THESIS ON
ON SOME ELASTICITY PROBLEMS OF FINITE MEMBERS SUBJECTED TO FUNCTIONAL LOADS

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(O.P. Sharma)

CERTIFICATE

This is to certify that the thesis entitled "On Some Elasticity Problems of Finite Members Subjected to Functional Loads", which is being submitted by Mr. O.P. Sharma for the award of Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, New-Delhi, is a record of bonafide research work. He has worked for the last two and a half years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirement of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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S Y N O P S I S

The thesis divided into four chapters comprises the solution of some problems in (i) Flexure of Beams and (ii) Stresses in Finite Cylinders subjected to Functional Loads. The closed form solutions for the Saint Venant's problem of flexure have been obtained using complex residue theory and Conformal-Mapping and applying Milne-Thomson method. Problems of three-dimensional Elasticity namely stresses in finite cylinders have been solved by the use of Galerkin Vectors and finite Fourier and Hankel transforms.

Chapter one - Introduction

Chapter two:

This chapter deals with Saint Venant's problem of flexure of isotropic, elastic beam of homogeneous material in the form of a prism or a cylinder. The importance in Engineering of the problem of the flexure of a prismatic beam is conspicuous by the considerable research which it has attracted and consequently the problem has been reduced to the determination of six canonical flexure functions. It can further be reduced to the determination of a single flexure function by conformally mapping the cross-section of the beam on a unit circle and using complex residue theory. We give a short account of the general theory and the basic equations. Simply connected cross-

sections have been dealt with. Considered in detail are the following problems:-

<u>Problem 1</u>: Flexure of a Regular Curvilinear Polygonal Section Beams: The expressions in closed form are obtained for the flexure function, moment, torsion function and centre of flexure for an isotropic elastic beam of a regular curvilinear polygonal section, in the form of a cylinder, whose region in the z-plane can be mapped conformally onto the interior of a unit circle in ζ-plane by the mapping function

$$z = \frac{c\zeta}{1 + m\zeta^n}\,;\ c > 0;\ n \geqslant 2,\ -1 \leqslant m(n-1) \leqslant 1$$

where c and m are real constants and n is a positive integer. In view of the closed form expressions every aspect of the problem can be studied. In particular the results for the following different shapes of cross-section are derived:

(a) Inverse of an ellipse $(n = 2,\ |m| < 1)$

(b) A circle of radius c $(m = 0 = n)$

<u>Problem 2</u>: Flexure of Beams whose cross-sections are bounded by certain quartic Curves: The closed form expressions have been obtained for the different functions for an isotropic homogeneous elastic cylinderical beam whose cross section is bounded by a quartic curve and whose region within its boundary in z-plane is mapped conformally onto the interior of a unit circle in ζ-plane by the mapping function

$$z = \frac{c\zeta}{1+n\zeta+m\zeta^2}, \quad c > 0$$

n and m are real parameters and satisfy the inequalities

$$-1 < m < 1 \quad \text{and} -2 < n < 2$$

Two cases when $n^2 = 4m$ and $n^2 \neq 4m$ have been considered in detail. Taking $n = 0$, the shape of the cross-section becomes a Booth's Lembiscate and the results for this case have been derived.

<u>Chapter Three</u>

This chapter deals with the determination of stresses and displacements in an isotropic, homogeneous, elastic and axisymmetric circular cylinder of finite dimensions and subjected to functional loads. Linear stress-strain relations have been assumed. Galerkin vector approach has been used to derive formulas for the stresses and displacements. A brief description of the general theory and basic equations is given. Following problems have been dealt in detail:

<u>Problem 1</u> Stresses in a Finite Cylinder resting on a Smooth Surface and subjected to functional loads: The displacement vector $\underline{u}$ is expressed in a Galerkin vector $\underline{U}$ and making its use in the basic equation of elasticity

$$G\left(\nabla^2 + \frac{1}{1-2\nu}\nabla \,\mathrm{div}\right)\underline{u} + \underline{F} = 0$$

(where symbols have their usual meaning and $\underline{F}$ is the body force).

the expressions for the stresses and displacements are derived after making use of an infinite series solution of the biharmonic equation $\nabla^4 \Psi = 0$. By the use of boundary conditions and finite Hankel and Fourier transforms an infinite set of simultaneous equations is set up which can be numerically solved.

Problem 4 and 5: Stresses in a Finite Cylinder resting on a Rough Surface and subjected to functional loads: This problem has also been solved with the Galerkin vector method. Two different positions of the cylinder have been discussed. In one case the lower surface of the cylinder is taken embedded in the rough surface on which it is resting whereas in the second case the cylinder rests freely on the resting pad. In both these problems two modes of load application are discussed. In one case the load is applied vertically on the upper surface of the cylinder i.e. $z = 0$ whereas in the second case it is applied horizontally along the face $z = 0$

Chapter Four

This chapter contains the tables and graphs of the results obtained by numerically solving the problems 3,4 and 5 of chapter three. The method discussed in chapter three is exemplified by numerically solving these problems on ICT 1909 Computer by taking length to radius ratios of the cylinder equal to 1 and 2. The following expressions for the load function have been considered:

(a) when the force acts vertically on the surface $z = 0$ the load function $f(r)$ is given by

(i) $$f(r) = q_0 , \quad 0 \leqslant r \leqslant \frac{a}{2}$$
$$= 0 , \quad \frac{a}{2} < r \leqslant a$$

(ii) $$f(r) = \sum_{n=1}^{\infty} k_n J_0(\alpha_n r), \quad 0 \leqslant r \leqslant a$$

(b) when the force acts horizontally along the face $z = 0$, the load function $g(r)$ is takne as

$$g(r) = \sum_{n=1}^{\infty} t_n J_1(\alpha_n r), \quad 0 \leqslant r \leqslant a.$$

Interpretations of the results are given with the help of tables and graphs.

C O N T E N T S