

**SENSITIVITY AND ROUND OFF NOISE  
IN  
DIGITAL FILTERS**

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CERTIFICATE

This is to certify that the thesis entitled 'SENSITIVITY AND ROUND OFF NOISE IN DIGITAL FILTERS' being submitted by Mrinmoy Bhattacharya to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a record of bonafide research work carried out by him under our supervision and guidance and in our opinion, it has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted to any other institute for the award of any degree or diploma.



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## ABSTRACT

The errors due to coefficient quantization and arithmetic roundoff in a recursive digital filter depend on the location of the poles in the  $z$ -plane. The poles and zeros of sharp cut-off filters are expected to be clustered around band edges in close proximity of the unit circle; this leads to an approximate relation between quantization errors and the bandwidth. Analysis of a few of the existing low sensitivity structures reveals inadequacy of these structures to encompass the entire ranges of bandwidth for such filters, for implementation with low sensitivity. Some new structures are presented in the thesis for removing this limitation. Some observations on common features generally associated with any low sensitivity structure, led to development of modified schemes for some of the existing and the new structures, whereby sensitivity and roundoff noise could be reduced further, by marginal extra computation.

For implementation of sharp cut-off bandpass/bandstop recursive filters with low sensitivity, a new approach is presented. This utilizes the transformation of sensitivity characteristics of a low sensitivity lowpass structure to an appropriate angular region on the unit circle. It is shown

that such an approach would realize bandpass/bandstop filters with low sensitivity associated with reduction in number of multiplications, while maintaining the sensitivity at the same level as that of the prototype lowpass filter.

The analysis of sensitivity in elemental second- and fourth-order sections for cascade realization of linear phase nonrecursive filters, with respect to the location of zeros, has been shown to yield similar results as in recursive filters. Some new structures for nonrecursive filters have been developed. It is shown that these structures, chosen appropriately for individual sections, would lead to a cascade realization with low sensitivity.

Analysis and simulation results on various filters implemented with the structures presented in the thesis are quite promising, and assert the utility of these structures for practical realization with low sensitivity.

## C O N T E N T S

	Page No.
Acknowledgement	... i
Abstract	... ii
Contents	... iv
List of Abbreviations and Symbols	... vi
 CHAPTER 1	
INTRODUCTION	... 1
1.1 Background	... 3
1.2 Outline of the Thesis	... 7
 CHAPTER 2	
SOME NEW STRUCTURES FOR REALIZATION OF RECURSIVE LOWPASS AND HIGHPASS FILTERS WITH LOW SENSITIVITY AND ROUND OFF NOISE	... 9
2.1 Introduction	... 9
2.2 New Structures	... 16
2.3 Sensitivity Analysis	... 23
2.4 Concluding Remarks	... 25
 CHAPTER 3	
MODIFIED SCHEMES FOR FURTHER REDUCTION OF SENSITIVITY AND ROUND OFF NOISE	... 27
3.1 Introduction	... 27
3.2 Modified Schemes	... 30
3.3 An Example	... 36
3.4 Performance Analysis	... 39
3.5 Some Comments on Realization of Filters.	... 50
3.6 Concluding Remarks	... 54
Appendix	... 55

CHAPTER 4	DESIGN OF BANDPASS AND BANDSTOP RECURSIVE FILTERS WITH LOW SENSITIVITY ...	64
	4.1 Introduction ...	64
	4.2 Some New Structures ...	69
	4.3 Sensitivity Characteristics of the various structures ...	82
	4.4 Performance Evaluation ...	87
	4.5 Some Additional Aspects ...	95
	4.6 Concluding Remarks ...	99
CHAPTER 5	CASCADE REALIZATION OF LINEAR PHASE NONRECURSIVE FILTERS WITH LOW SENSITIVITY ...	101
	5.1 Introduction ...	101
	5.2 Sensitivity Analysis for Normal Forms ...	102
	5.3 Some New Structures ...	104
	5.4 LTC Method of Reducing Sensitivity..	111
	5.5 Computational Considerations ...	111
	5.6 Performance Analysis ...	113
	5.7 Concluding Remarks ...	118
CHAPTER 6	CONCLUSIONS ...	119
	6.1 Main Results of the Thesis ...	119
	6.2 Scope of Further Research ...	121
	REFERENCES ...	126
	BIO-DATA ...	132