

# VARIABLE DIGITAL FILTERS

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in Electrical Engineering

by  
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CERTIFICATE

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The results contained in this thesis have not been submitted to any other institute for the award of any degree or diploma.

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## ABSTRACT

In many applications of digital filters, it is required to vary the cutoff frequency of a lowpass, or the bandwidth and centre frequency of a bandpass filter, through the variation of a small number of parameters, preferable one, in the structure. The design techniques for such variable filters make use of the spectral transformations which convert a prototype digital filter into another with different pass and stopbands. For variable IIR filter design, use is made of an allpass transformation; applied to an IIR prototype filter, however, this gives rise to delay free loops in the structure, which cannot be implemented digitally. For linear phase FIR variable cutoff filters, a first order cosine transformation has been used with success.

This thesis is concerned with the investigations of variable digital filters based on these two approaches. In the case of allpass transformation used with IIR prototypes, a simple and computationally efficient method is suggested to overcome the problem of delay free loops, and it is shown to be particularly useful when the IIR prototype filter is realized as a cascade or parallel combination of first and second order sections. An approach for the implementation of such variable IIR filters is also suggested.

Studies relating to the linear phase variable FIR filters start with a critical examination of the first order cosine

transformation and its advantages and disadvantages. A second order form of this transformation is derived which eliminates the drawbacks without increasing the computational load on the filter. Techniques are given for designing variable cutoff highpass, and variable bandwidth and centre frequency bandpass filters with this transformation. The transformation is extended to the design of two dimensional variable lowpass and bandpass FIR filters. A number of structures, including a new nested form, are studied with regard to coefficient sensitivity, output roundoff noise, computational efficiency and storage requirements.

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