

IMPROVING BRAIN ACTIVITY MAPPING THROUGH ENHANCED EEG SOURCE LOCALIZATION

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by

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THESIS CERTIFICATE

This is to certify that the thesis titled **IMPROVING BRAIN ACTIVITY MAPPING THROUGH ENHANCED EEG SOURCE LOCALIZATION, SUBMITTED TO IIT-D**, by **Anchal Yadav (2018CRZ8302)**, for the award of the degree of **Doctor of Philosophy**, is a bona fide record of the research work done by her under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: EEG; Second-order statistics; Covariance; Multivariate Fourier Decomposition Method; Sparse iterative covariance-based estimation approach; Likelihood-based Estimation of sparse parameters, MEMD, MFDM, MVMD, Graph Signal Processing

Localizing the active brain source using EEG signals is gaining a lot of interest from researchers. EEG signals are non-invasive, easy to capture, and economically feasible. Furthermore, these signals have better temporal resolution than MRI, CT scans, PET scans, etc. EEG signals have lower spatial resolution due to signal loss as it travels from the brain to electrode. So, with EEG source localization, the aim is to improve the spatial resolution of EEG signals.

In this work, second-order statistics have been proposed to enhance the aperture of the electrodes using the concept of virtual electrodes. With these virtual electrodes, a greater number of active sources can be estimated and localized with fewer electrodes.

New covariance-based methods are proposed for EEG source localization, like SPICE (Sparse Iterative Covariance-based Estimation) and LIKES (LIKelihood-based Estimation of Sparse parameters). These methods are robust to noise and improve the source localization results. Since EEG signals are highly affected by noise, decomposition-based methods like MEMD (Multivariate Empirical Mode Decomposition), MVMD (Multivariate Variational Mode Decomposition), and MFDM (Multivariate Fourier Mode Decomposition) have been used as a preprocessing step, and then the signals are localized. Further, by using these decompositions, the decomposed signal components corresponding to various sources are obtained. From these decomposed signal components, the source-related components are chosen. For the selection of the source-related components, various metrics like kurtosis, skewness, and entropy are studied and used.

Since EEG signals are multidimensional with information in time, frequency, trial, subject, etc. So in the next problem, the EEG source localization was done in the tensor domain. The various decomposition methods gave better results than directly applying source localization. So, these decomposition methods are used to obtain information in the frequency domain. Hence, the tensor is formed using information in channels, electrodes, and IMFs. This tensor is decomposed using the Canonical Polyadic Decomposition method (CPD), and the results are localized using the source-related spatial information used after decomposing. Till now, the connectivity of underlying sources has not been considered. So, in the next step, the connection of the sources is considered using graph signal processing. Using these

concepts, the modified optimization problem is obtained. This problem is solved using FISTA.

सार

कीवर्ड: ईईजी; द्वितीय-क्रम सांख्यिकी; सहप्रसरण; बहुभिन्नरूपी फूरियर विघटन विधि; विरल पुनरावृत्तीय सहप्रसरण-आधारित अनुमान दृष्टिकोण; विरल मापदंडों का संभावना-आधारित अनुमान, एमईएमडी, एमएफडीएम, एमवीएमडी, ग्राफ सिग्नल प्रोसेसिंग

ईईजी संकेतों का उपयोग करके सक्रिय मस्तिष्क स्रोत का स्थानीयकरण शोधकर्ताओं से बहुत रुचि प्राप्त कर रहा है। ईईजी संकेत गैर-आक्रामक, पकड़ने में आसान और आर्थिक रूप से व्यवहार्य हैं। इसके अलावा, इन संकेतों में एमआरआई, सीटी स्कैन, पीईटी स्कैन आदि की तुलना में बेहतर अस्थायी संकल्प है। ईईजी संकेतों में मस्तिष्क से इलेक्ट्रोड तक यात्रा करते समय संकेत के नुकसान के कारण कम स्थानिक संकल्प होता है। इसलिए, ईईजी स्रोत स्थानीयकरण के साथ, उद्देश्य ईईजी संकेतों के स्थानिक संकल्प में सुधार करना है।

इस कार्य में, आभासी इलेक्ट्रोड की अवधारणा का उपयोग करके इलेक्ट्रोड के एपर्चर को बढ़ाने के लिए द्वितीय-क्रम सांख्यिकी प्रस्तावित की गई है। इन आभासी इलेक्ट्रोड के साथ, कम इलेक्ट्रोड के साथ अधिक सक्रिय स्रोतों का अनुमान लगाया जा सकता है और स्थानीयकृत किया जा सकता है। EEG स्रोत स्थानीयकरण के लिए नए सहप्रसरण-आधारित तरीके प्रस्तावित किए गए हैं, जैसे SPICE (विरल पुनरावृत्तीय सहप्रसरण-आधारित अनुमान) और LIKES (विरल मापदंडों का LIKelihood-आधारित अनुमान)। ये तरीके शोर के लिए मजबूत हैं और स्रोत स्थानीयकरण परिणामों में सुधार करते हैं। चूंकि EEG सिग्नल शोर से अत्यधिक प्रभावित होते हैं, इसलिए MEMD (मल्टीवेरिएट एम्पिरिकल मोड डिक्ंपोजिशन), MVMD (मल्टीवेरिएट वैरिएशनल मोड डिक्ंपोजिशन) और MFDM (मल्टीवेरिएट फूरियर मोड डिक्ंपोजिशन) जैसे अपघटन-आधारित तरीकों का उपयोग प्रीप्रोसेसिंग चरण के रूप में किया गया है, और फिर सिग्नल स्थानीयकृत किए गए हैं। इसके अलावा, इन अपघटनों का उपयोग करके, विभिन्न स्रोतों के अनुरूप अपघटित सिग्नल घटक प्राप्त किए जाते हैं। इन विघटित सिग्नल घटकों में से, स्रोत-संबंधित घटकों का चयन किया जाता है। स्रोत-संबंधित घटकों के चयन के लिए, कर्टोसिस, तिरछापन और एन्ट्रॉपी जैसे विभिन्न मेट्रिक्स का अध्ययन और उपयोग किया जाता है।

चूंकि ईईजी सिग्नल समय, आवृत्ति, परीक्षण, विषय आदि में जानकारी के साथ बहुआयामी होते हैं। इसलिए अगली समस्या में, ईईजी स्रोत स्थानीयकरण टेंसर डोमेन में किया गया था। विभिन्न अपघटन विधियों ने सीधे स्रोत स्थानीयकरण को लागू करने की तुलना में बेहतर परिणाम दिए। इसलिए, इन अपघटन विधियों का उपयोग आवृत्ति डोमेन में जानकारी प्राप्त करने के लिए किया जाता है। इसलिए, चैनलों, इलेक्ट्रोड और आईएमएफ में जानकारी का उपयोग करके टेंसर का निर्माण किया जाता है। इस टेंसर को कैनोनिकल पॉलीएडिक अपघटन विधि (CPD) का उपयोग करके विघटित किया जाता है, और अपघटन के बाद उपयोग किए गए स्रोत-संबंधित स्थानिक जानकारी का उपयोग करके परिणामों को स्थानीयकृत किया जाता है। अब तक, अंतर्निहित स्रोतों की कनेक्टिविटी पर विचार नहीं किया गया है। इसलिए, अगले चरण में, स्रोतों के कनेक्शन को ग्राफ सिग्नल प्रोसेसिंग का उपयोग करके माना जाता है। इन अवधारणाओं

का उपयोग करके, संशोधित अनुकूलन समस्या प्राप्त की जाती है। इस समस्या को FISTA का उपयोग करके हल किया जाता है।

चूंकि ईईजी सिग्नल समय, आवृत्ति, परीक्षण, विषय आदि में जानकारी के साथ बहुआयामी होते हैं। इसलिए अगली समस्या में, ईईजी स्रोत स्थानीयकरण टेंसर डोमेन में किया गया था। विभिन्न अपघटन विधियों ने सीधे स्रोत स्थानीयकरण को लागू करने की तुलना में बेहतर परिणाम दिए। इसलिए, इन अपघटन विधियों का उपयोग आवृत्ति डोमेन में जानकारी प्राप्त करने के लिए किया जाता है। इसलिए, चैनलों, इलेक्ट्रोड और आईएमएफ में जानकारी का उपयोग करके टेंसर का निर्माण किया जाता है। इस टेंसर को कैनोनिकल पॉलीएडिक अपघटन विधि (CPD) का उपयोग करके विघटित किया जाता है, और अपघटन के बाद उपयोग किए गए स्रोत-संबंधित स्थानिक जानकारी का उपयोग करके परिणामों को स्थानीयकृत किया जाता है। अब तक, अंतर्निहित स्रोतों की कनेक्टिविटी पर विचार नहीं किया गया है। इसलिए, अगले चरण में, स्रोतों के कनेक्शन को ग्राफ सिग्नल प्रोसेसिंग का उपयोग करके माना जाता है। इन अवधारणाओं का उपयोग करके, संशोधित अनुकूलन समस्या प्राप्त की जाती है। इस समस्या को FISTA का उपयोग करके हल किया जाता है।

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ABBREVIATIONS

AP	Action Potential
PSP	Post-synaptic Potential
CNS	Central Nervous System
EEG	Electroencephalogram
BEM	Boundary Element Method
FEM	Finite Element Method
FDM	Finite Difference Method
MNE	Minimum Norm Estimate
WMNE	Weighted Minimum Norm Estimate
LORETA	Low-resolution electrical tomography
sLORETA	Standardized low-resolution brain electromagnetic tomography
LASSO	Least absolute Shrinkage and Selection Operator
SNR	Signal to Noise Ratio
FOCUSS	Focal Underdetermined System Solver
so-MNE	second-order Minimum Norm Estimate
so-sLORETA	second-order Low-resolution electrical tomography
so-LASSO	second-order Least absolute Shrinkage and Selection Operator
MSP	Multiple Sparse Priors
SPM	Statistical Parameter Mapping
EMD	Empirical Mode Decomposition
IMF	Intrinsic Mode Functions
EEMD	Ensemble Empirical Mode Decomposition
CEEMD	Complete Ensemble Empirical Mode Decomposition
MEMD	Multivariate Empirical Mode Decomposition
MVMD	Multivariate Variational Mode Decomposition
FDM	Fourier Decomposition Technique
FIBFs	Fourier Intrinsic Band Functions
ZPFB	Zero Phase Filter Bank
MFDM	Multivariate Fourier Decomposition Technique
SPICE	SParse Iterative Covariance-based Estimation
LIKES	LIKelihood-based Estimation of Sparse parameters
MSE	Mean Square Error
STFT	Short Term Frequency Transform
WT	Wavelet Transform

CPD	Canonical Polyadic Decomposition
BTD	Block Term Decomposition
NLS	Nonlinear Least Squares
CWT	Continuous Wavelet Transform
ALS	Alternating Least Squares

NOTATION

E	Electric Field
V_f	Electric Potential
B	Magnetic Potential
H	Magnetic Field
σ_0	conductivity
ϵ	vacuum permitivity
R_1, R_2, R_N	radius of concentric spheres
$\zeta_1, \zeta_2, \zeta_N$	tangential conductivities of concentric spheres
γ	angle between R_0 and R_e
D^p	Dipole Moment
α	R_0 (dipole location) and D^p
P_1, P_n	Legendre polynomials
D_r^p	Radial Dipole Moment
D_t^p	Tangential Dipole Moment
U^H	potential at electrode for homogeneous sphere
U^A	potential at electrode for anisotropic sphere
Y	matrix of potential at the electrode
G	Lead Field Matrix
D	Dipole moment
n	Noise vector
α	regularization term
I	Identity Matrix
β	is the angle between P_1 and P_n
R_{YY}	Covariance matrix of observed signals Y
N_e	Number of Electrodes
R_{DD}	Covariance matrix of dipole moment matrix D
r_y	vectorization vector of R_{YY}
p	vectorization of R_{DD}
G_{kron}	Kronecker product of matrix G
\hat{D}	estimated dipole strength
SNR	Signal to Noise Ratio
E_{loc}	Localization Error
T	Time samples
x_{est}	estimated source strength
x_{true}	Actual Sources strength
$y(t)$	potential at electrodes vector
γ_{li}	i^{th} IMF, FIBF, or VMF of L^{th} electrode
r_l	residue of decomposed methods EMD, VMD, or FDM
ϵ_l	residual noise plus Gaussian noise of l^{th} electrode
χ_i	dipole source strength corresponding to i^{th} independent component

p_i	contains i^{th} decomposed components of all electrodes
$Kurt$	Kurtosis
S_i	Skewness
en_i	entropy
G_1, G_2	Graph
V_1, V_2	set of nodes of EEG graph
$E1, E2$	Edges of EEG graph
W_1, W_2	Weights of graphs
$R(D)$	regularization term
$Diff$	heat kernel difference