

A MINIMUM VARIANCE APPROACH TO SYSTEM
IDENTIFICATION AND ITS APPLICATIONS

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CERTIFICATE

I am satisfied that the thesis presented by Rajendra K.P. Bhatt is worthy of consideration for the award of the degree of Doctor of Philosophy and is a record of the original bonafide research work carried out by him under my guidance and supervision and that the results contained in it have not been submitted in part or full to any other University or Institute for award of any degree/diploma.

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ABSTRACT

The main conclusion to be drawn from this thesis is that in using Kalman filter for identification of linear systems, there is no need to linearize an augmented (state and parameters) nonlinear system as is done in the both extended and linearized Kalman filter methods. Two algorithms are developed, each based on Kalman filter, for estimation of unknown parameters in a linear system. After showing their convergence and simulation performance, applications of system identification in construction of adaptive observer and adaptive state estimator for unknown linear systems, control of unknown linear systems and design of digital filters are considered.

To develop the two parameter estimation algorithms the system equations in state space form are first transformed into an equivalent difference eqn. representation. For the first algorithm, only those measurements which are $(n+1)$ sampling intervals apart (for n th order system), are used for parameter estimation. This data spacing is done to whiten a correlated noise in a rearranged form of equivalent recursive difference

equation representation of unknown system. A direct application of Kalman filter to this rearranged form with data spacing, results in an estimator which is shown to yield biased parameter estimates. The bias is corrected and the first algorithm is obtained which yields asymptotically unbiased parameter estimates converging in mean square sense to true values. For the second algorithm, without using data spacing, a Kalman filter is derived from basic results which yields suboptimal minimum variance estimates of unknown system parameters from the equivalent recursive difference equation representation. The estimates are shown to be converging to true values in the mean square sense and upper bound of estimation error covariance is also made directly available. Among the extensions of this approach, time varying parameters, estimation of noise parameters, identification using bounded state estimators for coloured noise and multi-input/output systems are considered. Several simulation examples are presented and it is seen that the second algorithm yields minimum estimation error and fastest identification.

Finally, applications of system identification in (i) construction of adaptive observer and adaptive state estimator for unknown linear deterministic and stochastic systems respectively,

(ii) control of unknown linear systems and
(iii) design of digital filters, are considered. The first application uses intermediate results from parameter estimation algorithms presented earlier while the third application is based on the fact that in both identification and filter design unknown parameters have to be evaluated from the given response.

The thesis concludes with a summary of results and few suggestions for further work.

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