NUMERICAL INVESTIGATIONS OF THE FLOW PAST AXISYMMETRIC SURFACE PROTUBERANCES USING A NEW COMPUTATIONAL SCHEME

by

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1. Particle velocity upwinding
2. High speed flow

Certificate

This is to certify that the thesis entitled **Numerical Investigations of the Flow Past Axisymmetric Surface Protuberances using a new Computational Scheme** being submitted by **Mr. Adnan Qamar** to the **Indian Institute of Technology Delhi,** for the award of the degree of Doctor of Philosophy in Department of Applied Mechanics is a bonafide research work carried out by him under my supervision and guidance. The research work presented in this thesis has not been submitted in parts or in full to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

The present study is an endeavour to elucidate three aspects of high-speed flow. Firstly, a new computational scheme called the Particle Velocity Upwinding (PVU) Scheme has been proposed for the computations of hyperbolic systems. Secondly, a numerical investigation, utilizing the proposed PVU scheme for the problem of axisymmetric surface protuberance mounted on a cylindrical base with hemispherical nose has been carried out. Finally, the Proper Orthogonal Decomposition (POD) technique has been coupled with direct interpolation and extrapolation and the marching extrapolation procedure to yield an efficient and economical tool for predicting the steady high-speed flows.

The proposed PVU scheme utilizes the classical upwinding approach by employing the fluid particle velocity or the entropy wave speed at the cell interface to establish the upwind direction for the convective term. The governing equations are split into two flux vectors, the convective flux vector and the non-convective flux vector. The cell face convective fluxes are obtained from a first order or a second order upwind biased interpolation, depending on whether the cell under consideration lies in the vicinity of a discontinuity or in a region of steep gradients in the solution. The discontinuities or regions of steep gradients are detected by employing a smoothness indicator function. The non-convective terms are discretized using the values available at the grid node instead of using the cell face values, thus resulting in a staggering type of approach. The spatial discretization strategy is combined with that of a predictor-corrector type of time integration technique, which results in

overall stable methodology for computing flows governed by hyperbolic conservation laws.

Further, utilizing the proposed PVU scheme the problem of the axisymmetric surface protuberance of triangular and trapezoidal cross-section has been carried out in a curvilinear coordinate system. The investigations are conducted at a fixed free stream Reynolds number of 1.8×10^4 and Prandtl number of 0.72. Effects of three parameters namely, the protuberance height, the inflow Mach number and the protuberance cross-section on the global and local flow field are scrutinized.

The spatial global pattern exhibits a strong bow shock formation in front of the spherical nose. The Mach number variation only affects this bow shock in term of shock strength and shock deflection. The variations in protuberance height and the cross section does not affect this bow shock. In the local region of the protuberance, a weak oblique shock wave is formed in the upstream region. Expansion waves from the sharp corners of the protuberances and a recompression wave in the downstream of the protuberance are formed. The upstream and the downstream region of the protuberance, for some cases, are dominated by the presence of separated regions. The weak oblique shock wave formed in front of the protuberance interacts with the boundary layer and causes shock wave boundary layer interaction. Since the oblique shock is weak in nature it does not cause separation, it only results in increase in pressure levels near the protuberance for all the cases investigated.

An increase in Mach number results in the movement of the local oblique shock towards the protuberance. Therefore, the upstream separation region also narrows down. As the Mach number is increased in the downstream region, the recompression wave moves closer to the protuberance. Also, the base separation is drastically affected by an increase in Mach number. The base separation decreases rapidly with an increase in Mach number and for some cases (triangular protuberance, high Mach number cases) the separated region virtually vanishes. For small height (H>0.02) there is no separation of flow at the front and in base of protuberance irrespective of Mach number and protuberance cross-section. For H>0.02 the flow separation takes place ahead of the protuberance and at the base of the protuberance. For all the protuberance height and cross-sections heat transfer reversal takes place at approximately Mach 5.

When the protuberance cross-section is changed, the strength of the oblique shock formed upstream of the trapezoidal protuberance is more as compared to the oblique shock wave formed in upstream of the triangular protuberance. In the upstream and the downstream region, the heat transfer rates are almost comparable for both the protuberances. On the protuberance surface when the heat transfer is from body to fluid then higher heat transfer rate is observed for the triangular protuberance. However, when heat transfer is reversed the heat transfer is more for the trapezoidal protuberance. Separation in front and base of the protuberance is less for the triangular protuberance. Since the oblique shock strength is more for trapezoidal protuberance, the peak pressure is higher for the trapezoidal protuberance. The induced drag for the triangular protuberance is less as compared to the trapezoidal protuberance for all Mach number and heights.

Finally, the POD approach is coupled with the interpolation-extrapolation and the marching extrapolation procedure to predict the steady supersonic flow field by

ensembling the data obtained by the numerical simulations. The procedure developed, utilizes an ensemble data for a given parametric variation and the goal of the proposed POD method is to predict the steady flow field that has not been included in the parametric variations. The problem of axisymmetric surface mounted triangular protuberance is utilized to demonstrate the efficiency and the accuracy of the proposed strategy. The Mach number and the protuberance height are used as the two parameters, for which the POD snapshots are colleted by varying these parameters in the specified interval.

The direct POD interpolation procedure performs accurately and efficiently when the flow field is reconstructed for the parameter whose value lies inside the snapshot ensembling interval. In order to predict the flow field outside the snapshot ensembling interval the direct POD extrapolation procedure is used. It is found that, in order to predict the complete supersonic steady state flow-field accurately, using POD interpolation or extrapolation procedure, the RMS deviations in the components of the reconstructed field from the ensembled data should be less than 1%. The direct POD extrapolation procedure is quite accurate near the ensembling interval but the predictions are inaccurate as one moves farther from the ensembling interval. Thus, a Marching POD extrapolation (MPODE) procedure is suggested which results in ameliorating the overall RMS error by 50% compared with the direct POD extrapolation procedure.

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