

REPRESENTATIONS OF TWO MODE
PARA-BOSE OPERATORS

by

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DEDICATED TO THE LOVING
MEMORY OF MY FATHER

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