

STUDIES IN OPTIMAL CONTROL AND ESTIMATION  
IN TIME-DELAY SYSTEMS

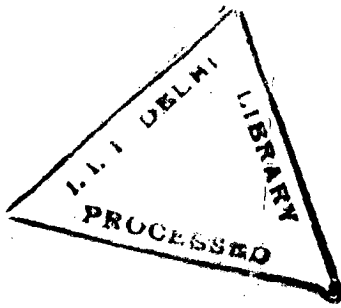
by

J. MISHRA

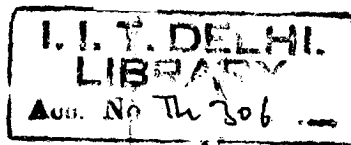
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Department of Electrical Engineering,  
Indian Institute of Technology, Delhi,  
New Delhi-110 029.

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## ABSTRACT

This thesis is concerned with the study of the problems relating to optimal control, and state and parameter estimation of time-delay systems. Discrete dynamic programming, Innovations approach and Moore-Penrose pseudoinverse techniques have been used as the basic tool for the same.

The optimal control via dynamic programming has been discussed in chapters II and III of this thesis. A particular approach to the problem has been to discretize the system equation as well as the criterion functional and then augment the state vector so as to convert the system with delay to one without delay. Known results can then be applied to the augmented system. However, the method suffers from high dimensionality due to state augmentation. In the methods proposed in this thesis, it has been tried to circumvent this difficulty.

The first approach is to apply discrete dynamic programming directly to the discrete version of the problem. It is found that the algorithm thus obtained has several advantages over that based on state augmentation.

The second method utilizes the concept of state augmentation and the results derived there from. It is found that there is considerable misuse of computer

storage and time in unnecessary calculations involving null and identity submatrices. Hence, in the second method proposed, the matrices in the final results are decomposed so as to isolate the zeros and identity submatrices. An operation count proves the efficiency of the method.

Although sufficient work has been done to develop the equations of optimal linear filtering, prediction and smoothing for linear discrete systems, comparatively little has been reported for time-delay systems. Some results in this field have been reported by Kwakernaak [28], Biswas and Mahalanabis [31], [32], Priemer and Vacroux [29], Farooq and Mahalanabis [30], and RajaRao and Mahalanabis [33]. Biswas and Mahalanabis have used the method of state augmentation and Kalman filtering technique to derive equations for time-delay systems with added complexities. In chapter IV of this thesis the technique of innovations approach has been used to develop algorithms for the following cases:

- (i) when the measurement noise is colored, and
- (ii) when the state and measurement noises are correlated.

It has been observed that the approach is more elegant and direct. The first algorithm [36] is more general in the sense that it covers all the previous results. The second algorithm gives a simpler algorithm than that proposed by Raja Rao and Mahalanabis.

Just like state estimation, very little has been reported regarding the parameter estimation in time delay systems. Hsia [40] has applied Kalman's least-squares method for identifying the parameters of a continuous system by identifying the parameters of an equivalent discrete model from the input output data. The method employs essentially the generalized inverse technique which can be used only off-line.

It may be observed here that the recursive generalized inverse technique that has been used for non-time-delay systems cannot be used directly to time-delay systems, because the matrix whose generalized inverse is found does not always possess full rank. In chapter V of this thesis an on-line algorithm has been proposed based on iterative pseudoinverse technique to identify the parameters of a discrete system or the equivalent discrete system when the system is continuous.

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