

**VIBRATION CONTROL AND MECHANICS-DRIVEN
MACHINE LEARNING TECHNIQUES FOR
HIGH-SPEED RAILWAY BRIDGES**

SUSMITA PANDA



**DEPARTMENT OF CIVIL ENGINEERING
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MACHINE LEARNING TECHNIQUES FOR
HIGH-SPEED RAILWAY BRIDGES**

by

SUSMITA PANDA

DEPARTMENT OF CIVIL ENGINEERING

submitted

**in fulfillment of requirements of degree of Doctor of Philosophy
to the**



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DEDICATION

I would like to dedicate my thesis to my family, for their endless support and boundless love for making me the person I am today.

CERTIFICATE

This is to certify that the thesis titled **Vibration control and mechanics-driven machine learning techniques for high-speed railway bridges**, submitted by Ms. Susmita Panda, to the Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by her under our supervision and guidance. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Bappaditya Manna
Professor
Dept. of Civil Engineering
IIT Delhi, 110016

Dr. Arnab Banerjee
Assistant Professor
Dept. of Civil Engineering
IIT Delhi, 110016

New Delhi
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ABSTRACT

KEYWORDS: moving load analysis; mode-superposition method ; surrogate models ; vibration control; inertial amplifier; resonator; negative stiffness; genetic algorithm

The analysis of design parameters of high-speed railway bridges in critical locations such as canal, road/railway crossings is an important aspect of bridge engineering. Nevertheless, there are concerns about the structural safety and stability of high-speed rail bridges, particularly those with simple support, due to their elevated speeds. The situation can become even more complex in two-way loading scenarios, where two trains may cross at varying speeds.

In the present work, the analysis of bridges under one-way as well as two-way loading is analyzed under two support conditions: simply support (SS) and elastically-supported (ES). For the sake of brevity, the moving load models were considered and the bridge is idealized as an Euler Bernoulli beam of uniform cross-section. Three cases, namely single moving load, series of moving loads at equal spacing (HSLM-B), and as per conventional train configuration (HSLM-A) recommended in Eurocode1: EN 1991-2 (2003), are considered. A non-dimensional framework is developed to systematically analyze the resonance and cancellation phenomena under various incidental load and bridge parameters. Such analysis reduces the parametric space and time for computation. Additionally, a parametric investigation was comprehensively addressed to determine the effect of these parameters on the dynamic response of the bridge.

However, to bypass the high-fidelity models, surrogate models were developed in conjunction with the non-dimensional framework. A predictive surrogate model employing feed-forward multi-layered neural networks is adopted to conserve computational effort and reduce the amount of expertise needed in the preliminary evaluation of dynamic responses. Furthermore, the robustness and efficiency of the best-fitting surrogate models were com-

pared, and a user interface was designed to assess responses in accordance with the standards outlined in Eurocode EN 1990:2002+A1:2005 (E) and HSR Rating Code (TZY2014-232 2014).

In order to address the vehicle-induced vibrations caused by the passage of high-speed trains, the vibration absorbers adopted in the present analysis are resonator, inertial amplifier (IA), and negative-stiffness inertial amplifier. Various configurations of the negative-stiffness inertial amplifier vibration absorber (NSIAVA) were explored by adjusting the mass ratio (χ), including the negative-stiffness based vibration absorber (NSBVA) and inertial amplifier-based vibration absorber (IABVA). The optimization of the absorber parameters was performed using a genetic algorithm.

Comparative studies between the resonator and IA revealed that IA, with a lower mass ratio and higher values of the speed parameter (η) and inter-spatial distance between loads (ϵ), proved more effective in reducing vibration amplitudes due to significant mass amplification. Furthermore, IA was found to achieve vibration suppression while maintaining lightweight characteristics, distinguishing it from resonators.

The NSIAVA exhibited distinct mass characteristics, with NSBVA possessing negative mass and IABVA having positive mass. However, as the mass tuning ratio increased from 0.6 to 0.9, the effect of negative mass diminished, eventually becoming positive at $\chi = 1$ for IABVA. At resonance condition, and varying main mass ratios, an efficiency of 70-90% is achieved for different configurations of vibration absorber, particularly $\chi=0$ to 0.8. Additionally, the outcomes demonstrate that the Negative Stiffness-Based Vibration Absorber (NSBVA) outperforms the Inertial Amplifier-Based Vibration Absorber (IABVA).

Therefore, special consideration is required when designing high-speed railway bridges to account for resonance as well as cancellation speeds that occur under specific train load configurations. These resonance speeds are associated with particular bridge lengths. Engineers can select this cancellation speed for critical bridge locations. Furthermore, optimizing the parameters of vibration absorbers makes it possible to minimize vibration induced by vehicle resonance.

संक्षेप

नहर, सड़क/रेलवे क्रॉसिंग जैसे महत्वपूर्ण स्थानों पर हाई-स्पीड रेलवे पुलों के डिजाइन मापदंडों का विश्लेषण पुल इंजीनियरिंग का एक महत्वपूर्ण पहलू है। फिर भी, हाई-स्पीड रेल पुलों की संरचनात्मक सुरक्षा और स्थिरता के बारे में चिंताएं हैं, विशेष रूप से सरल समर्थन वाले पुलों की, उनकी ऊंची गति के कारण। दो-तरफा लोडिंग परिदृश्यों में स्थिति और भी जटिल हो सकती है, जहां दो ट्रेनें अलग-अलग गति से पार कर सकती हैं।

वर्तमान कार्य में, एक-तरफा के साथ-साथ दो-तरफा लोडिंग के तहत पुलों का विश्लेषण दो समर्थन स्थितियों के तहत किया जाता है: बस समर्थन (एसएस) और इलास्टिक-समर्थित (ईएस)। संक्षिप्तता के लिए, मूविंग लोड मॉडल पर विचार किया गया और पुल को समान क्रॉस-सेक्शन के यूलर बर्नौली बीम के रूप में आदर्श बनाया गया है। तीन मामलों, अर्थात् एकल गतिमान भार, समान दूरी पर गतिमान भार की श्रृंखला (एचएसएलएम-बी), और यूरोकोड1: एन 1991-2 (2003) में अनुशंसित पारंपरिक ट्रेन कॉन्फिगरेशन (एचएसएलएम-ए) के अनुसार, पर विचार किया जाता है। विभिन्न आकस्मिक भार और पुल मापदंडों के तहत अनुनाद और रद्दीकरण घटना का व्यवस्थित रूप से विश्लेषण करने के लिए एक गैर-आयामी ढांचा विकसित किया गया है। इस तरह के विश्लेषण से गणना के लिए पैरामीट्रिक स्थान और समय कम हो जाता है। इसके अतिरिक्त, पुल की गतिशील प्रतिक्रिया पर इन मापदंडों के प्रभाव को निर्धारित करने के लिए एक पैरामीट्रिक जांच को व्यापक रूप से संबोधित किया गया था।

हालाँकि, उच्च-निष्ठा मॉडल को बायपास करने के लिए, सरोगेट मॉडल को गैर-आयामी ढांचे के संयोजन में विकसित किया गया था। कम्प्यूटेशनल प्रयास को संरक्षित करने और गतिशील प्रतिक्रियाओं के प्रारंभिक मूल्यांकन में आवश्यक विशेषज्ञता की मात्रा को कम करने के लिए फ्रीड-फ़ॉरवर्ड बहु-स्तरित तंत्रिका नेटवर्क को नियोजित करने वाला एक पूर्वानुमानित सरोगेट मॉडल अपनाया जाता है। इसके अलावा, सर्वोत्तम-फिटिंग सरोगेट मॉडल की मजबूती और दक्षता की तुलना की गई, और यूरोकोड ईएन 1990:2002+ए1:2005 (ई) और एचएसआर रेटिंग कोड (टीजेडवाई2014) में उल्लिखित मानकों के अनुसार प्रतिक्रियाओं का आकलन करने के लिए एक यूजर इंटरफेस डिजाइन किया गया था। -232 2014).

हाई-स्पीड ट्रेनों के गुजरने के कारण होने वाले वाहन-प्रेरित कंपन को संबोधित करने के लिए, वर्तमान विश्लेषण में अपनाए गए कंपन अवशोषक अनुनादक, जड़त्वीय एम्पलीफायर (आईए), और नकारात्मक-कठोरता जड़त्वीय एम्पलीफायर हैं। द्रव्यमान अनुपात (χ) को समायोजित करके नकारात्मक-कठोरता जड़त्वीय एम्पलीफायर कंपन अवशोषक (एनएसआईएवीए) के विभिन्न विन्यासों का पता लगाया गया, जिसमें नकारात्मक-कठोरता आधारित कंपन अवशोषक (एनएसबीवीए) और जड़त्वीय एम्पलीफायर-आधारित कंपन अवशोषक (आईएबीवीए) शामिल हैं। अवशोषक मापदंडों का अनुकूलन एक आनुवंशिक एल्गोरिथ्म का उपयोग करके किया गया था।

रेज़ोनेटर और IA के बीच तुलनात्मक अध्ययन से पता चला कि IA , कम द्रव्यमान अनुपात और गति पैरामीटर (η) के उच्च मान और भार (ε) के बीच अंतर-स्थानिक दूरी के साथ, कंपन आयाम को कम करने में अधिक प्रभावी साबित हुआ। महत्वपूर्ण जन प्रवर्धन के कारण। इसके अलावा, आईए को हल्की विशेषताओं को बनाए रखते हुए कंपन दमन प्राप्त करने के लिए पाया गया, जो इसे अनुनादकों से अलग करता है।

एनएसआईवीए ने विशिष्ट द्रव्यमान विशेषताओं का प्रदर्शन किया, जिसमें एनएसबीवीए में नकारात्मक द्रव्यमान और आईबीवीए में सकारात्मक द्रव्यमान था। हालाँकि, जैसे-जैसे द्रव्यमान ट्यूनिंग अनुपात 0.6 से बढ़कर 0.9 हो गया, नकारात्मक द्रव्यमान का प्रभाव कम हो गया, अंततः IABVA के लिए $\chi=1$ पर सकारात्मक हो गया। अनुनाद स्थिति और अलग-अलग मुख्य द्रव्यमान अनुपात पर, कंपन अवशोषक के विभिन्न विन्यासों के लिए 70-90% की दक्षता हासिल की जाती है, विशेष रूप से $\chi=0$ से 0.8 तक। इसके अतिरिक्त, परिणाम दर्शाते हैं कि नकारात्मक कठोरता-आधारित कंपन अवशोषक (एनएसबीवीए) जड़तीय एम्पलीफायर-आधारित कंपन अवशोषक (आईबीवीए) से बेहतर प्रदर्शन करता है।

इसलिए, विशिष्ट ट्रेन लोड कॉन्फ़िगरेशन के तहत होने वाली प्रतिध्वनि के साथ-साथ रद्दीकरण गति को ध्यान में रखते हुए हाई-स्पीड रेलवे पुलों को डिजाइन करते समय विशेष विचार की आवश्यकता होती है। ये अनुनाद गति विशेष पुल की लंबाई से जुड़ी हैं। महत्वपूर्ण पुल स्थानों के लिए इंजीनियर इस रद्दीकरण गति का चयन कर सकते हैं। इसके अलावा, कंपन अवशोषक के मापदंडों को अनुकूलित करने से वाहन अनुनाद से प्रेरित कंपन को कम करना संभव हो जाता है।

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List of Abbreviations

MoU	Memorandum of understanding
JICA	Japan International Cooperation Agency
DLF	Dynamic load factor
FEM	Finite Element Modeling
MCFM	Moving Constant Force Model
MHFM	Moving Harmonic Force Model
MMM	Moving mass model
MSDMM	Moving spring-damping-mass model
TAVBM	Two-axle vehicle-bridge model
TBDIM	Train bridge dynamic interaction model
TTBDIM	Train track bridge dynamic interaction model
AASHTO	American Association of State Highway and Transportation Officials
EADA	Equivalent additional damping approach
BFGS	Broyden Fletcher Goldfarb Shanno
VTSI	Vehicle-track-structure interaction
VMD	Variational mode decomposition
IMFs	Intrinsic mode functions
VMD	Variational mode decomposition
CNN	Convolutional neural network
FFNN	Feed forward neural network
LSTM	Long short-term memory
VBI	Vehicle-bridge interaction
RSM	Response Surface Method
LAW-RSM	Linear Adaptive Weighted RSM
NARX	Non-linear autoregressive with exogenous input
STMD	Single tuned mass damper

MTMD	Multiple tuned mass damper
PTMD	Pounding tuned mass damper
MTMFD	Multiple tuned mass friction dampers
IANMNS	Inertial Amplifier Negative Mass Negative Stiffness
NSIABI	Negative stiffness inertial amplifier base isolator
IATMD	Inertial amplifier tuned mass damper
FD	Friction dampers
TMFDs	Tuned mass friction dampers
VDs	Viscous dampers
VEDs	Viscoelastic dampers
MR	Magnetorheological dampers
ANFIS	Adaptive neuro-fuzzy inference system technique
PPF	Positive position feedback control
SMC	Sliding mode control
MDOF	Multi-degree of freedom
SPT	Sequential programming technique
ANN	Artificial neural network
MIMO	Multiple input multiple outputs
SS	Simply-support
E-B	Euler-Bernoulli
ES	Elastically-supported
HSLM	High-speed load model
IA	Inertial amplifier
MSE	Mean square error
RMS	Root mean square error
EVS	Explained variance score
SDR	Statistical displacement response
SEM	Standard error of mean
MOE	Margin of error
IQR	Interquartile range
NSBVA	Negative-stiffness based vibration absorber
NSIABVA	Negative-stiffness based inertial amplifier based vibration absorber

IABVA	Inertial-amplifier based vibration absorber
SAIVM-TMD	Semi-active independent variable mass tuned mass damper
SABFD	Semi-Active Bypass Fluid Damper device

List of Symbols

EI	flexural rigidity
ρA	mass per unit area
C_b	damping coefficient of the bridge
P_V	forces due to the moving load
$\tilde{W}_b$	dynamic displacement of the bridge
Q_b	dimensionless displacement of the bridge
x	position on the bridge
t	time taken by moving force to travel the bridge
$\delta(x)$	dirac-delta function
v	speed of the moving vehicle
L	length of bridge
d	axle spacing
d/v	time lag
P_0	gravitational weight of the running train
$P(t)$	step function
t_{kd}	time delay of k^{th} load in HSLM-B model
t_{kD}	time delay of k^{th} load in HSLM-A model
$\tilde{Q}_{bj}(t)$	generalised coordinate
$\phi_j(x)$	shape function of j^{th} mode
$\tilde{V}$	shear force
$\tilde{M}$	bending moment
M	dimensionless bending moment
V	dimensionless shear force
ω	frequency of vibration of moving load
ω_{bj}	natural frequency of the beam for j^{th} mode
ω_b	natural frequency of the beam for first mode

ζ_b	damping coefficient
C_c	critical damping constant
$Q_{bj}(x, t)$	dimensionless displacement of beam
Δ_{st}	static constant for displacement
Δ_{sf}	static constant for shear force
Δ_{bm}	static constant for bending moment
a	dimensionless length over the bridge
τ	dimensionless time
η	speed parameter
$N - bogies$	number of bogies
ϵ_d	ratio of the distance between loads to the length of the bridge in HSLM-B model
ϵ_D	ratio of the distance of a single bogie to the length of the bridge
ω_d	damped frequency of vibration of the beam
M_v	mass matrix
C_v	damping matrix
K_v	stiffness matrix
t_{kD} and $\bar{\tau}_D$	time taken and corresponding time lag of the k^{th} bogie separated by ϵ_D
t_b and $\bar{\tau}_b$	time taken and corresponding time lag of loads P_{t1} and P_{t2} separated by ϵ_d
t_c and $\bar{\tau}_c$	time taken and corresponding time lag of loads P_{t1} and P_{t3} separated by ϵ_c
K	stiffness of elastic bearing
κ	stiffness ratio of beam to elastic bearings
ω_s	fundamental frequency of ES beam
EI	flexural rigidity
R	speed parameter ratio of two trains
t_{os}	time delay between two trains
T_{os}	non-dimensional time delay between two trains
Δ_{ES}	static deflection of ES beam
S_r	resonance speed parameter
S_c	cancellation speed parameter
Q_{bloc}	location of maximum displacement of beam
R_r	resonance speed parameter ratio between two trains
R_c	cancellation speed parameter ratio between two trains

ϵ_{min}	minimum value of the distance between two loads to the length of the beam
ϵ_{max}	maximum value of the distance between two loads to the length of the beam
M_a	auxiliary masses of inertial amplifier
M_r	main mass of inertial amplifier
K_r	stiffness of inertial amplifier
θ	inertial angle
f_x	inertial forces in x-direction
f_y	inertial forces in y-direction
M_{ia}	total effective mass of inertial amplifier
X_a	deflection of main mass in x-direction
Y_a	deflection of main mass in y-direction
M_{eff}	effective mass
K_{eff}	effective stiffness
m	mass ratio
γ_t	frequency ratio
m_r	main mass ratio
Y_r	displacement of the vibration absorber
y_r	dimensionless displacement of the vibration absorber
ζ_{xt}	location of vibration absorber
χ	mass tuning ratio
M_n	lateral masses of vibration absorber
K_a	lateral stiffness of vibration absorber
γ	lateral frequency ratio
ω_n	natural frequency of laterally connected system in vibration absorber
M_t	total lateral mass of vibration absorber