

# **FORCE - EXCITED VIBRATION OF NONLINEAR PANELS**

**DINESH JHA**

**DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
JUNE 1981**

**FORCE - EXCITED VIBRATION OF**  
**NONLINEAR PANELS**

by

**DINESH JHA**

**Thesis submitted to the Indian Institute of Technology  
Delhi, for the award of the degree of**

**DOCTOR OF PHILOSOPHY**

**Department of Mechanical Engineering  
Indian Institute of Technology, Delhi**

**June 1981**

## ABSTRACT

The present report is a work on nonlinear vibration of thin panels. It deals with the theoretical evaluation and experimental validation of the response of panels subjected to harmonic and random excitations.

1. Panels subjected to harmonic excitation :-

The theoretical investigation deals with the analysis of the large amplitude vibration of nonlinear panels subjected to harmonic excitation acting at the centre point of panel. For derivation of equations of motion the panel is considered to be acted upon by distributed force of general type. Expressions for strain energy and kinetic energy of the vibrating nonlinear panel are obtained. Hamilton's principle is used to obtain the Euler-Lagrange equations of motion of the panel. Galerkin's technique is applied to the Euler-Lagrange equations to reduce them to a simple differential equation of Duffing type. The Duffing type of equation is solved by perturbation technique to obtain the response of the panels. Experimental rig is fabricated and thin panels of rectangular and triangular shapes have been tested on the rig to obtain the response when excited by harmonic excitation. Theoretical and experimental results agree closely. Jump phenomenon is observed in the amplitude-frequency response curves during a transition period of a few seconds.

(ii)

2. Panels subjected to Random Excitation :-

Theoretical analysis has been done for the random vibration of nonlinear panels. Study is made of the vibration characteristics of nonlinear panels subjected to broad-band type of excitation at a point. Analytical solution of response is also obtained for the nonlinear panels subjected to non-white-noise type of random excitation. Equivalent linearisation technique is employed for obtaining a linear model of the panel. The response of the linearized panels subjected to above excitations are obtained. Experimental investigation is conducted in the laboratory to obtain the response of the panels when subjected to random excitation at a point. Experimental and theoretical results agree reasonably.

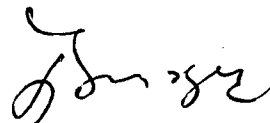
3. Panels subjected to Turbulent Boundary Layer Excitation:-

Response analysis is done for panels subjected to turbulent boundary layer excitations. Principle of equivalent linearisation is applied to the nonlinear panels to obtain a linear model. Response of linearized panels subjected to turbulent boundary layer excitation is obtained. In the present work the model of pressure fluctuations in the turbulent boundary layer as proposed by M.K. Bull [38] has been used for obtaining the response of the panels. Experimental set up is prepared to mount the panels in the subsonic wind tunnel for measurement of response. Experimental and theoretical results have been compared. Multi-mode response analysis has been done in this area.

CERTIFICATE

This is to certify that the Thesis entitled 'Force-Excited Vibration of Nonlinear Panels' being submitted by Mr. Dinesh Jha to the Indian Institute of Technology, Delhi, for the award of the Degree of 'Doctor of Philosophy' in Mechanical Engineering is a record of bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for the submission of this Thesis, which has reached the requisite standard.

The results contained in this Thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.



(Kailash Nath Gupta)  
Professor  
Department of Mechanical Engineering  
Indian Institute of Technology, Delhi,  
New Delhi-110016.

ACKNOWLEDGEMENTS

It is with great pleasure that the author wishes to express his gratefulness to Dr. K.N. Gupta, Professor of Mechanical Engineering Department for supervising the work on this thesis. His inspiring guidance, valuable discussions and constant encouragement helped the author to complete this work.

The author wishes to thank the Department of Applied Mechanics for permission to work on the wind tunnel of Gas Dynamics Laboratory.

Thanks are due to the staff of Vibrations and Instrumentation Laboratory, Departmental Workshop, Departmental Instruments Repairing Cell, I.D.D. Centre and Computer Centre for their help and cooperation.

The financial support for the present work provided by the Ministry of Education and Social Welfare, Government of India New Delhi under Quality Improvement Programme is gratefully acknowledged.

A special word of thanks goes to Mr. V.P. Gulati for typing this manuscript.

Finally the author offers his heartfelt thanks to his near ones for patiently enduring difficulties till the completion of this research project.

*Dinesh Jha*  
(DINESH JHA)

CONTENTS

		<u>PAGE NO.</u>
	ABSTRACT	i
	CERTIFICATE	iii
	ACKNOWLEDGEMENTS	iv
	CONTENTS	v
	TERMINOLOGY	ix
	LIST OF FIGURES	xii
	LIST OF TABLES	xvi
CHAPTER-1	INTRODUCTION	1
CHAPTER-2	LITERATURE SURVEY	4
2.1	Free Vibrations of Panels	4
2.2	Panels Subjected to Harmonic Excitation	6
2.3	Panels Subjected to Random Excitation	7
2.4	Panels Excited by Turbulent Boundary Layer Pressure Fluctuations	13
2.5	Outline of the Present Investigation	17
CHAPTER-3	RESPONSE OF NONLINEAR PANELS TO HARMONIC EXCITATION	20
3.1	Introduction and Assumptions	20
3.1.1	Equations of Motion of Nonlinear Panel	22
3.2	Application of Galerkin's Technique	31

	<u>PAGE NO.</u>
3.3	Applications 33
3.3.1	Clamped Rectangular Panel 33
3.4	Solution of Duffing Type Equation by Perturbation Technique 37
3.5	Clamped Isotropic Right Triangular Panel 42
3.6	Experimental Investigation 45
3.6.1	Experimental Set-up and Instrumentation 45
3.6.2	Experimental Procedure 46
3.7	Results and Discussions 48
CHAPTER-4	SOME BASIC CONCEPTS OF A RANDOM FIELD PROCESS 70
4.1	General Random Field Process 70
4.2	Stationary Random Field Process 73
4.3	Homogeneous Random Field Process 74
4.4	Stationary and Homogeneous Random Field Process 75
4.5	Field Process Variance Function and Field Process Spectral Density 76
4.6	Series Development of a Random Field Process 78
4.7	Stationary Random Loading of Panels 81
4.8	Excitation Process Considered in the Present Work 84
4.8.1	Single-Point Random White-Noise Excitation 84
4.8.2	Turbulent Boundary Layer Excitation 85

		<u>PAGE NO.</u>
CHAPTER-5	RESPONSE OF NONLINEAR PANEL TO POINT LOAD RANDOM EXCITATION	87
5.1	General Concepts	87
5.2	Technique of Linearisation	88
5.3	Evaluation of Mean Square Response	90
5.3.1	White-Noise Excitation	92
5.3.2	Non-White-Noise Excitation	93
5.3.3	Relationship between Spectral Density of Exciting Force and the Response of the Panel	94
5.4	Experimental Investigation	95
5.4.1	Experimental Set-up and Instru- mentation	95
5.4.2	Experimental Procedure	96
5.5	Results and Discussions	97
CHAPTER-6	RESPONSE OF A NONLINEAR PANEL EXCITED BY A RANDOM FIELD PROCESS	110
6.1	Introduction	110
6.2	Method of Solution	111
6.2.1	Equations of Motion in Principal Co-ordinates	111
6.2.2	Application of Equivalent Linearisation Technique	120
6.2.3	Evaluation of Mean Square Response	121
6.2.4	Types of Excitation Process	124

	<u>PAGE NO.</u>
6.3 Experimental Investigation	139
6.3.1 Experimental Set-up and Instrumentation	139
6.3.2 Experimental Procedure	141
6.4 Results and Discussions	143
CHAPTER-7----- CONCLUSIONS AND SCOPE FOR FUTURE WORK	175
REFERENCES	179
APPENDIX I	190

.....