

GEOMETRICALLY NONLINEAR SMART BEAM FORMULATION AND APPLICATIONS

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GEOMETRICALLY NONLINEAR SMART BEAM FORMULATION AND APPLICATIONS

by

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Submitted

in fulfillment of the requirements for the award of the degree of Doctor of Philosophy

to the



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*Dedicated to
My Parents
M S Khan & Musarrat Jahan*

CERTIFICATE

This is to certify that the thesis entitled “**GEOMETRICALLY NONLINEAR SMART BEAM FORMULATION AND APPLICATIONS**” being submitted by Mr. P M G Bashir Asdaque to Indian Institute of Technology Delhi for the award of degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

With advancements in modern technologies, self-monitoring and autonomous control are challenging tasks, especially for crucial structures in the civil, military, and aviation industries. For such challenges, piezoelectric materials are highly efficient, both in actuation and sensing capabilities. In this work, the 'smart' beams are confined to the piezoelectric-based slender beam-like structures. Splitting of the original 3D nonlinear problem for 'smart' beams results in a linear 2D cross-sectional analysis and a geometrically exact 1-D smart beam analysis over the reference line. The major challenge in the research of modeling any piezoelectric transducer lies in capturing the coupling effects, both electromechanical and structural coupling.

The present work begins with developing an 'electromechanically coupled mixed variational formulation' to meet the future design challenges of piezoelectric-based slender beam-like sensors. One-dimensional finite element equations are derived based on this formulation, capable of dealing with sensor-beam problems undergoing large displacements and rotations. An extra electrical degree of freedom is added to the sensor elements. Later, the approach is extended to deal with multiphysics problems having both actuation and sensing portions on the same beam. Though 3D finite element analysis can handle such geometrically nonlinear and complex multiphysics problems, they are computationally expensive. At the same time, reduced models like the present one maintain simplicity and acceptable accuracy while being computationally efficient.

FE computational code has been developed on the FORTRAN platform and can study both static and dynamic problems. Newton-Raphson method is used to deal with geometric nonlinearity. The stiffness and mass matrices are obtained directly from the already developed

cross-sectional analysis tools for smart beams. Initially, the results obtained are validated with the benchmark problems of large displacement and rotations of passive beams, active and sensing beams present in the literature. Validation of mechanical and electrical outputs from 3D FEM software only proves the efficacy of the code. The run-time comparison tables provided prove the computational efficiency. Later, the Eigenvalue analysis in open and close circuit conditions of sensory beams have been done. Also, the Newmark method is used to generate the time marching-based dynamic behavior. Follower force condition is also studied, and an example of dynamic instability is also explored, depicting the versatility of the present theory.

After that, numerical strategies dealing with complex structures having multiple, distinct sensor and active links have been provided. These multiphysics-multilink structures can be thought of as many smart beams joining at particular connection points, such as truss and satellite bae-type linkages. Finally, a complete chapter is dedicated to the study of rotating smart beams. The effect of rotation on several parameters of a rotating sensor is studied. The impact of simultaneous actuation and rotation is also studied. This platform's future version can be utilized for more practical applications like monitoring and active control of flexible smart multilink structures like swept wings, multi-bae space structures, and rotating helicopter blades.

सार

आधुनिक तकनीकों में प्रगति के साथ, स्व-निगरानी और स्वायत्त नियंत्रण चुनौतीपूर्ण कार्य हैं, विशेष रूप से नागरिक, सैन्य और विमानन उद्योगों में महत्वपूर्ण संरचनाओं के लिए। ऐसी चुनौतियों के लिए, पीजोइलेक्ट्रिक सामग्री सक्रियता और संवेदन क्षमताओं दोनों में अत्यधिक कुशल हैं। इस काम में, 'स्मार्ट' बीम पीजोइलेक्ट्रिक-आधारित पतला बीम जैसी संरचनाओं तक ही सीमित हैं। 'स्मार्ट' बीम के लिए मूल तीन-आयामी नॉन-लीनियर समस्या के विभाजन के परिणामस्वरूप एक लीनियर दो-आयामी क्रॉस-सेक्शनल विश्लेषण और संदर्भ रेखा पर एक ज्यामितीय सटीक एक-आयामी स्मार्ट बीम विश्लेषण होता है। किसी भी पीजोइलेक्ट्रिक ट्रांसड्यूसर के मॉडलिंग के शोध में प्रमुख चुनौती विद्युत और संरचनात्मक दोनों के युग्मन प्रभावों को पकड़ने में निहित है।

वर्तमान कार्य पीजोइलेक्ट्रिक-आधारित पतला बीम-जैसे सेंसर की भविष्य की डिजाइन चुनौतियों को पूरा करने के लिए 'इलेक्ट्रोमैकेनिकल रूप से युग्मित मिक्स्ड वेरिएशनल फॉर्मूलेशन' विकसित करने के साथ शुरू होता है। एक आयामी परिमित तत्व समीकरण इस फॉर्मूलेशन के आधार पर व्युत्पन्न होते हैं, जो बड़े विस्थापन और घूर्णन से गुजरने वाली सेंसर-बीम समस्याओं से निपटने में सक्षम होते हैं। सेंसर तत्वों में स्वतंत्रता की एक अतिरिक्त विद्युत डिग्री जोड़ी जाती है। बाद में, एक ही बीम पर एक्चुएशन और सेंसिंग दोनों हिस्से वाले मल्टीफिजिक्स समस्याओं से निपटने के लिए दृष्टिकोण का विस्तार किया गया है। हालांकि तीन-आयामी परिमित तत्व विश्लेषण ऐसी ज्यामितीय अरेखीय और जटिल बहु-भौतिकी समस्याओं को संभाल सकता है, वे कम्प्यूटेशनल रूप से महंगे हैं। साथ ही, वर्तमान मॉडल जैसे रेडूसड मॉडल कम्प्यूटेशनल रूप से कुशल होने के दौरान सादगी और स्वीकार्य सटीकता बनाए रखते हैं।

एफई कम्प्यूटेशनल कोड को फोरट्रान प्लेटफॉर्म पर विकसित किया गया है और यह स्थिर और गतिशील दोनों समस्याओं का अध्ययन कर सकता है। न्यूटन-रैफसन विधि का प्रयोग ज्यामितीय अरेखिकता से निपटने के

लिए किया जाता है। स्मार्ट बीम के लिए पहले से विकसित क्रॉस-सेक्शनल विश्लेषण टूल से कठोरता और द्रव्यमान मैट्रिक्स सीधे प्राप्त किए जाते हैं। प्रारंभ में, प्राप्त परिणामों को साहित्य में मौजूद निष्क्रिय बीम, सक्रिय और सेंसिंग बीम के बड़े विस्थापन और घूर्णन की बेंचमार्क समस्याओं के साथ मान्य किया जाता है। श्री-डी ऍफ़-इ-एम् सॉफ्टवेयर से मैकेनिकल और इलेक्ट्रिकल आउटपुट का सत्यापन केवल कोड की प्रभावकारिता को साबित करता है। प्रदान की गई रन-टाइम तुलना तालिकाएं कम्प्यूटेशनल दक्षता को साबित करती हैं। बाद में, संवेदी पुंजों की ओपन और क्लोज सर्किट स्थितियों में आइगन-वैल्यू विश्लेषण किया गया है। साथ ही, टाइम मार्चिंग-आधारित गतिशील व्यवहार उत्पन्न करने के लिए न्यूमार्क पद्धति का उपयोग किया जाता है। अनुयायी बल की स्थिति का भी अध्ययन किया जाता है, और वर्तमान सिद्धांत की बहुमुखी प्रतिभा को दर्शाते हुए गतिशील अस्थिरता का एक उदाहरण भी खोजा जाता है।

उसके बाद कई विशिष्ट सेंसर और सक्रिय लिंक वाले जटिल संरचनाओं से निपटने वाली संख्यात्मक रणनीतियां प्रदान की गई हैं। इन मल्टीफिजिक्स-मल्टीलिनक संरचनाओं को विशेष कनेक्शन बिंदुओं पर शामिल होने वाले कई स्मार्ट बीम के रूप में माना जा सकता है, जैसे टूस और सैटेलाइट बे-टाइप लिंकेज। अंत में, एक पूरा अध्याय घूर्णन स्मार्ट बीम के अध्ययन के लिए समर्पित है। घूर्णन सेंसर के कई मापदंडों पर घूर्णन के प्रभाव का अध्ययन किया जाता है। एक साथ क्रिया और घूर्णन के प्रभाव का भी अध्ययन किया जाता है। इस प्लेटफॉर्म के भविष्य के संस्करण का उपयोग अधिक व्यावहारिक अनुप्रयोगों के लिए किया जा सकता है जैसे कि लचीले स्मार्ट मल्टीलिनक संरचनाओं की निगरानी और सक्रिय नियंत्रण जैसे स्वेप्ट विंग्स, मल्टी-बे स्पेस स्ट्रक्चर और रोटेटिंग हेलीकॉप्टर ब्लेड।

CONTENTS

CERTIFICATE	i
ACKNOWLEDGEMENTS	iii
ABSTRACT	v
LIST OF FIGURES	xiii
LIST OF TABLES	xvii
NOMENCLATURES	xx
CHAPTER 1 INTRODUCTION	1
1.1 Preface	1
1.2 Literature review	3
1.2.1 Smart beam models	4
1.2.2 Cross-sectional analysis	6
1.2.3 One-dimensional beam formulation	7
1.2.4 Follower forces and dynamic instability	7
1.2.5 Smart multilink structures	9
1.2.6 Recent developments and exciting applications of piezo-based smart structures	11
1.3 Motivation and objectives	12
1.4 Organization of the thesis	14
1.5 Discussion	16
CHAPTER 2 FORMULATION	17

2.1 Introduction	17
2.2 Sensor beam	18
2.2.1 Mixed variational formulation for sensor beam	18
2.2.2 Finite element equations	28
2.3 Smart beams composed of actuators and sensors	32
2.3.1 Mixed formulation	33
2.3.2 Finite element equations	37
2.4 Discussion	41
CHAPTER 3 STATIC ANALYSIS: BENCHMARK VALIDATIONS FOR LARGE DISPLACEMENTS, ACTUATION AND SENSING	42
3.1 Introduction	42
3.2 Benchmark tests for large displacements: passive beam	42
3.3 Actuator model validation	47
3.4 Sensor model validation	50
3.5 Discussion	54
CHAPTER 4 DYNAMIC ANALYSIS OF SMART COMPOSITE SENSOR BEAMS	56
4.1 Introduction	56
4.1.1 Newmark time marching	56
4.1.2 Eigenvalue problem	58
4.2 Results	59

4.2.1 Eigenvalue analysis	59
4.2.2 Transient dynamic response: dead periodic load	65
4.2.3 Transient dynamic response: periodic follower load	71
4.2.4 Dynamic stability analysis	75
4.3 Discussion	76
CHAPTER 5 SMART MULTILINK STRUCTURES: ACTIVE AND SENSOR LINKS	77
5.1 Introduction	78
5.2 Clamped-clamped swept beam	80
5.3 Tree type	82
5.4 Closed four-link model	85
5.5 Response due to harmonic actuation	89
5.6 Impulse actuation	92
5.7 Discussion	93
CHAPTER 6 ROTATING SMART BEAMS	94
6.1 Introduction	94
6.2 Effect of rotation: sensor beam	94
6.3 Effect of actuation: rotating smart multilink structures	97
6.4 Discussion	104
CHAPTER 7 CONCLUSION & FUTURE SCOPE OF WORK	104
7.1 Introduction	105

7.2 Major contribution	105
7.3 Observations	106
7.4 Future scope of work	107
APPENDICES	109
REFERENCES	125
PUBLICATIONS FROM THIS RESEARCH WORK	133
VITA	135

LIST OF FIGURES

Figure 1.1 The decoupling of slender 3D continuum into 1D beam and 2D cross-sectional analysis	2
Figure 1.2 Dead (dashed red line) and follower forces	8
Figure 2.1 High fidelity smart beam modeling	17
Figure 2.2. The representation of the reference frames	18
Figure 2.3 Finite element description of the i^{th} element	28
Figure 2.4 Multiphysics straight beam	33
Figure 3.1 Elastic curves of the passive cantilever under increasing end shear load	44
Figure 3.2 Elastic curves of the cantilever beam under increasing end moment	45
Figure 3.3 A three-layer actuator beam	48
Figure 3.4 Transverse displacement along the normalized length of the beam (substrate: Aluminium) (CH: Chee et al., 1999; SR: Saravanos and Heyliger, 1995, EL: Elshafei and Alraies, 2013)	50
Figure 3.5 Transverse displacement along the normalized length of the beam (substrate: T300/934[0] ⁰)	50
Figure 3.6 Convergence test	51
Figure 3.7 Electric potential along the length compared to the literature (Neto: Augusta Neto et al., 2009; CH: Chee et al., 1999; SR: Saravanos and Heyliger, 1995)	52
Figure 3.8 Deflection plot of sensor composite beam	52
Figure 3.9 Transverse displacement of the slender smart beam	53
Figure 3.10 Electric voltage output for the slender smart beam	54
Figure 4.1 Convergence test	62

Figure 4.2 Short (left) and open circuit (right) conditions	64
Figure 4.3. Forcing function on the tip of the sensor cantilever beam	66
Figure 4.4 Linear time history	68
Figure 4.5 Nonlinear time history	68
Figure 4.6 Transverse displacement obtained at 0.5 m from the fixed end	69
Figure 4.7 Transverse displacement obtained at 0.2m from the free end	70
Figure 4.8 Electric potential obtained at 0.5 m from the fixed end	70
Figure 4.9 Transverse displacement obtained at the free end when forcing frequency is 10Hz	70
Figure 4.10 Electric potential obtained from the 1 st element near the fixed end when forcing frequency is 10Hz	71
Figure 4.11 Time history validation of the present work with the ABAQUS for follower dynamic Load	73
Figure 4.12 Transient time responses for dead and periodic follower forces	74
Figure 4.13 Imaginary and real eigenvalues for open and short circuit condition flutter phenomena (Inset has zoomed picture at the critical point)	75
Figure 5.1 Schematic of multilink system	79
Figure 5.2 Clamped-clamped swept smart beam	80
Figure 5.3 Validation of the deformed structure	81
Figure 5.4 Electric potential obtained from the sensor link	82
Figure 5.5 Rotation parameter along the normalized length of the sensor link	82
Figure 5.6 Deformed shape of clamped-free-clamped tree model (scaled: 100x)	83
Figure 5.7 Voltage obtained from sensor links (starting from the common point of all the links)	83

Figure 5.8 Rotation parameter (about a_2) for sensor links in the clamped-free-clamped model	84
Figure 5.9 Mechanical deformation in the clamped-clamped-clamped tree model (scaled: 1000x)	84
Figure 5.10 Voltage produced by sensor links (starting from the connection point of all the links)	85
Figure 5.11 Schematic of the four-link smart model	86
Figure 5.12 Piezoelectric layer arrangement in ABAQUS for in-plane actuation (Joint 2)	86
Figure 5.13 Deformed shape validation (deformation scaled: 50x)	87
Figure 5.14 Validation of voltage produced at sensor links 1 & 2 (SL1 & SL2).	87
Figure 5.15 Rotation parameters for sensor links 1 & 2 (SL1 & SL2).	88
Figure 5.16 Out-of-plane actuation	88
Figure 5.17 Electric potential obtained from sensor links 1 & 2 (SL1 & SL2).	89
Figure 5.18 Time history of the point lying on mid of the beam axis of actuator.	90
Figure 5.19 Time history of the junction of actuator and sensor links.	90
Figure 5.20 The voltage output from the 5 th electrode of the sensor link.	91
Figure 5.21 The voltage output from the 10 th electrode on the sensor link	91
Figure 5.22 The voltage output from the 17 th electrode on sensor link	91
Figure 5.23 Schematic of swept beam under direct voltage loading (left), and applied voltage for 0.00001 s (right)	92
Figure 5.24 Axial and transverse displacements at the connection point/joint	92
Figure 5.25 Axial and transverse displacements at the free tip of the swept smart beam	93
Figure 6.1 Sketch of a rotating sensor beam	94

Figure 6.2 Steady-state behavior of smart beam under various rotating speed (Electric Potential vs angular speed)	96
Figure 6.3 Steady-state behavior of smart beam under various rotating speed (Moment at the root/fixed end vs angular speed)	96
Figure 6.4 Steady-state behavior of smart beam under various rotating speed (Rotation parameter at the free end vs angular speed)	96
Figure 6.5 Steady-state behavior of smart beam under various rotating speed (transverse displacement vs angular speed)	97
Figure 6.6 Schematic of a rotating swept smart beam	97
Figure 6.7 Rotation parameter at the mid-point of the actuator link	98
Figure 6.8 Transverse displacement at the mid-point of the actuator link	98
Figure 6.9 Rotation parameter at the free tip of sensor link	99
Figure 6.10 Transverse displacement at the free end of the sensor link	99
Figure 6.11 Rotation parameter at the mid-point of the actuator link	99
Figure 6.12 Transverse displacement at the mid-point of the actuator link	100
Figure 6.13 Rotation parameter at the free tip of sensor link	100
Figure 6.14 Transverse displacement at the free end of the sensor link	100
Figure 6.15 Rotation parameter at the mid-point of the actuator link	101
Figure 6.16 Transverse displacement at the mid-point of the actuator link	101
Figure 6.17 Rotation parameter at the free tip of sensor link	101
Figure 6.18 Transverse displacement at the free end of the sensor link	102
Figure 6.19 Rotation parameter at the mid-point of the actuator link	102
Figure 6.20 Transverse displacement at the mid-point of the actuator link	102

Figure 6.21 Rotation parameter at the free tip of sensor link	103
Figure 6.22 Transverse displacement at the free end of the sensor link	103

LIST OF TABLES

Table 2-1 Terms expressed in various frames of references.	28
Table 2-2 Shape functions.	40
Table 3-1. Material and geometric properties of homogeneous passive beams	42
Table 3-2. Axial (U) and transverse (W) deflections of the cantilever tip under the influence of end shear force	43
Table 3-3. Axial (U) and transverse (W) tip deflections of the cantilever beam under the influence of end moment	45
Table 3-4. Error percentage $\left\{\left(\frac{ S_{ze-Present} }{S_{ze}}\right)\times 100\right\}$ in Axial (U) and transverse (W) tip deflections of the cantilever beam under the influence of end shear load	46
Table 3-5. Error percentage $\left\{\left(\frac{ S_{ze-Present} }{S_{ze}}\right)\times 100\right\}$ in Axial (U) and transverse (W) tip deflections of the cantilever beam under the influence of end moment	46
Table 3-6. Geometric and material data of three-layer actuator beam	47
Table 3-7. Cross-sectional values	49
Table 3-8. Comparison of time consumption for the present model and ABAQUS simulation for increased slenderness.	54
Table 3-9 Summary of the analysis	54
Table 4-1. Validation of natural frequencies (Aluminium bonded with a single layer of PZT4)	60
Table 4-2. Geometric and material data of three-layer composite beam	62
Table 4-3. Natural frequencies (transverse bending) for open circuit condition obtained from the present method and the ABAQUS	63

Table 4-4. Natural frequencies (lateral bending) for open circuit condition obtained from the present method and the ABAQUS	63
Table 4-5. Open and short circuit: natural frequencies (transverse bending).	64
Table 4-6. Open and short circuit: natural frequencies (lateral bending).	64
Table 4-7. Time consumption for the present model and ABAQUS simulation.	69
Table 4-8. Time consumption for sinusoidal follower & dead loading for transient time histories recorded for a second.	72
Table 4-9. Critical flutter load and the flutter frequency for SC and OC condition	76
Table 5-1. Hierarchical description of terms involved	78
Table 5-2. Computational time comparison for the computer system and the present code	81

NOMENCLATURES

a_i	Global body attached frame
b_i	Undeformed reference frame
B_i	Deformed reference frame
Π	Electromechanical enthalpy
Γ	Second-order 3-D strain tensor
\mathcal{E}	One dimensional strain (comprising of both force and moment strains)
ϕ	Electrical potential
E	Electric field
r	Locus of the centroid of the cross-section in the undeformed state
\hat{r}	The position vector of any arbitrary material point in the undeformed cross-section
R	Locus of undeformed material points (r) in the deformed configuration
\hat{R}	The position vector of any arbitrary material point in the deformed cross-section
γ	One-dimensional force-strains
κ	One-dimensional moment-strains
S	Stiffness matrix

I	Mass matrix
V	Linear velocity in deformed frame
v	Linear velocity in undeformed frame
Ω	Angular velocity in deformed frame
ω	Angular velocity in undeformed frame
P	Linear momentum
H	Angular momentum
F	Internal force vector
M	Internal moment vector
u	Displacement vector in undeformed frame
θ	Rodrigues parameter
δ	Variational symbol
U	One-dimensional strain energy
K	One-dimensional kinetic energy
$\overline{\delta W}$	Virtual work equivalent
$\overline{\delta q}$	Virtual displacement vector
$\overline{\delta \psi}$	Virtual rotation vector

$\overline{\delta(\)}$ Overbar indicates ‘not an exact variation’ of any function

Subscripts

- \square_{Ac} Terms related to actuator
- \square_{Sn} Terms related to Sensor
- \square_a Terms represented in the body-attached frame
- \square_b Terms represented in the undeformed frame
- \square_B Terms represented in the deformed frame

Superscripts

- \square^{ba} The transformation from frame ‘a’ to ‘b’
- \square^{Bb} The transformation from frame ‘b’ to ‘B’
- \square^T Transpose of that matrix
- $(\square)'$ Derivative in x_1
- $(\dot{\square})$ Derivative in time (Temporal derivative)
- $(\tilde{\square})$ Tilde operator or cross-product operator