

**THE THEORETICAL ANALYTICAL SOLUTIONS OF
PIEZOTHEROELASTIC RESPONSE OF
PLATE AND SHELLS**

by

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Submitted

in fulfilment of the requirements of

the degree of

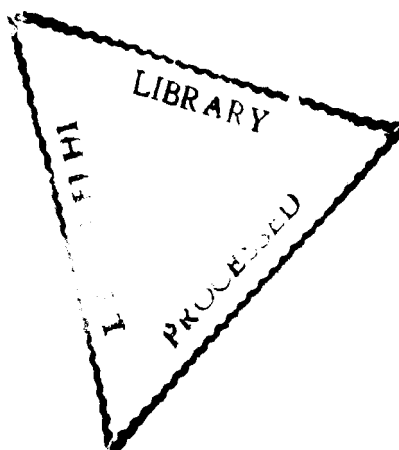
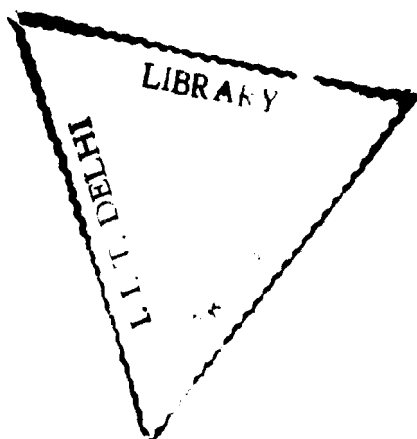
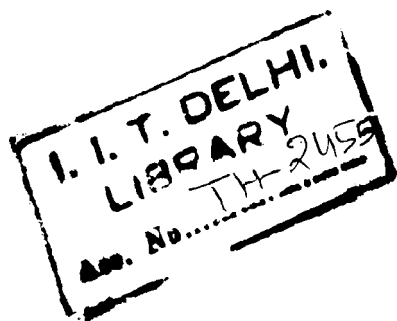
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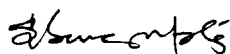
Dedicated to

*my father Late Sachindra Kapuria
whose ideas and thoughts have always inspired me*

CERTIFICATE

This is to certify that the thesis entitled "Three Dimensional Analytical Solutions for Piezothermoelastic Response of Plates and Shells" being submitted by Mr. Santosh Kapuria to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

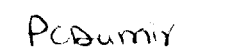

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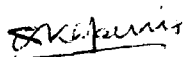
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ABSTRACT

Three dimensional (3D) exact analytical solutions for static piezothermoelastic response of some piezoelectric and hybrid (cross-ply substrate with piezoelectric layers) plates and shells are presented. The 3D strain-displacement relations, electric field-potential relations, thermal, force and charge equilibrium equations and the coupled piezothermoelasticity constitutive equations, are solved subject to the exact satisfaction of the boundary conditions of the structure and the interface conditions in the case of multilayered structure. The solution also involves determination of the unknown extraneous charge densities at the interfaces where prescribed potential or potential difference is applied. The 3D solutions are used to assess two dimensional (2D) uncoupled classical lamination theory (CLT and CST), and first order shear deformation theory (FSDT) of plates and shells, using Flugge's approximations for the latter. The 2D solutions are used in the 3D equilibrium equations and strain-displacement relation to obtain transverse stresses and an improved value of deflection. The inverse problems of estimating the pressure or temperature field from the measured potential difference are solved. The effect of geometrical parameters is ascertained on through-the-thickness variations of various entities, and the feasibility of reducing the deflection and stresses by proper actuation is studied. Effectiveness of various actuation schemes for shape control is examined for rectangular plates and cylindrical panels. Orthorhombic materials of crystal class $mm2$ / class 6mm are considered.

Plane strain solution for cylindrical bending and 3D solution of simply supported rectangular plates are presented using analytical solution of the governing differential equations with constant coefficients. Levy-type solution is also presented for rectangular plate with two edges simply supported using a mixed formulation of uncoupled FSDT and CLT. The plane strain solution of cylindrical bending of simply supported cylindrical panels is presented using analytical solution of Euler-Cauchy form of governing equations. The 3D solutions of simply supported, finite cylindrical shell under axisymmetric and general load are presented, wherein the governing differential equations with variable coefficients are solved by the modified Frobenius method using a product of an exponential function and a power series. The 3D axisymmetric solutions are presented for (1) a finite, transversely isotropic, axially polarised, piezoelectric cylindrical shell, (2) a transversely isotropic piezoelectric clamped circular plate and (3) a hybrid clamped circular isotropic elastic plate bonded to a transversely isotropic piezoelectric plate using potential function method.

It is concluded from the results that the effect of the thickness of the structure on the through-the-thickness distributions of various entities depends on the nature of load and the geometry. The distributions tend to be piecewise linear or uniform as the number of layers increases or the structure becomes thin. The potential in the actuator of the hybrid structure has almost linear variation across its thickness. The deflection and the maximum stress in the substrate due to thermal and pressure load can be reduced by suitable actuation. An actuated layer is most effective for shape control, when placed on the loaded surface of the structure. The error in 2D theories is less for the mechanical load than the thermal or potential load.

TABLE OF CONTENTS

CERTIFICATE	i
ACKNOWLEDGEMENTS	ii
ABSTRACT	iii
LIST OF PUBLICATIONS ABSTRACTED FROM THIS THESIS	iv
TABLE OF CONTENTS	v
LIST OF FIGURES	viii
1 INTRODUCTION	1
1.1 Preface	1
1.2 General solutions for piezoelectric media	2
1.3 3D solutions for rectangular plates	2
1.4 2D theories for hybrid rectangular plates	4
1.5 3D solutions for cylindrical shells	6
1.6 Objectives of this thesis	8
1.7 Cylindrical bending of flat panel	10
1.8 Rectangular plate	10
1.9 Circular cylindrical panel in cylindrical bending	11
1.10 Finite circular cylindrical shell under axisymmetric load	12
1.11 Finite circular cylindrical shell under general load	13
1.12 Axially polarised finite circular cylindrical shell	13
1.13 Piezoelectric clamped circular plate under axisymmetric load	13
1.14 Clamped circular hybrid plate under axisymmetric load	14
2 CYLINDRICAL BENDING OF FLAT PANEL	15
2.1 Introduction	15
2.2 Governing equations for 3D solution	15
2.3 General solution of 3D governing equations	19
2.4 3D solution for direct and inverse problems	23
2.5 3D results for piezoelectric panel	24
2.6 3D results for hybrid panel	34
2.7 Governing equations for FSDT	43
2.8 Solution for FSDT	45
2.9 Solution for CLT	46
2.10 Results of FSDT and CLT	47
2.11 Conclusions	50

3	RECTANGULAR PLATE	52
3.1	Introduction	52
3.2	Governing equations for 3D solutions	53
3.3	Solution of 3D governing equations	55
3.4	Shape control of rectangular plate	59
3.5	Governing equations for 2D solutions	66
3.6	Levy-type solution	67
3.7	Results for 2D solutions	70
3.8	Conclusions	77
4	CIRCULAR CYLINDRICAL PANEL IN CYLINDRICAL BENDING	79
4.1	Introduction	79
4.2	Governing equations for 3D solution	80
4.3	General solution of 3D governing equations	83
4.4	Solution for direct and inverse problems	87
4.5	3D results for piezoelectric panel under electromechanical load	88
4.6	3D results for piezoelectric panel under thermoelectric load	96
4.7	3D results for hybrid panel under electromechanical load	103
4.8	3D results for hybrid panel under thermoelectric load	110
4.9	Governing equations for FSDT	116
4.10	Governing equations for CST	118
4.11	Solution for FSDT and CST	119
4.12	Results for FSDT and CST	120
4.13	Conclusions	127
5	FINITE CIRCULAR CYLINDRICAL SHELL UNDER AXISYMMETRIC LOAD	128
5.1	Introduction	128
5.2	Governing equations	128
5.3	General solution of governing equations	131
5.4	Solution of eigenvalue problem	135
5.5	Solution for arbitrary constants	136
5.6	Results for piezoelectric shell under electromechanical load	137
5.7	Results for piezoelectric shell under thermoelectric load	148
5.8	Results for hybrid shell under electromechanical load	156
5.9	Results for hybrid shell under thermoelectric load	163
5.10	Conclusions	172
6	FINITE CIRCULAR CYLINDRICAL SHELL AND SHELL PANEL UNDER NONAXISYMMETRIC LOAD	173
6.1	Introduction	173
6.2	Governing equations for 3D solution	173
6.3	General 3D solution	175

6.4	3D solution for direct and inverse problems	178
6.5	3D results for shell under electromechanical load	180
6.6	3D results for shell under thermoelectric load	187
6.7	3D results for hybrid shell with PZT-5A actuator	194
6.8	3D results for panel under electromechanical load	201
6.9	3D results for panel under thermoelectric load	208
6.10	Governing equations for FSDT	213
6.11	Governing equations for CST	215
6.12	Solution for FSDT and CST	216
6.13	2D results for electromechanical load	218
6.14	2D results for thermoelectric load	227
6.15	Conclusions	237
7	AXIALLY POLARISED FINITE CIRCULAR CYLINDRICAL SHELL	239
7.1	Introduction	239
7.2	Governing equations	239
7.3	General solution of governing equations	240
7.4	Boundary conditions for axially restrained shell	241
7.5	Solution for axially restrained shell	242
7.6	Solution of inverse piezothermoelastic problems	245
7.7	Numerical results	247
7.8	Conclusions	251
8	PIEZOELECTRIC CLAMPED CIRCULAR PLATE	
	UNDER AXISYMMETRIC LOAD	252
8.1	Introduction	252
8.2	Governing equations	252
8.3	General solution for infinite circular plate	253
8.4	Algebraic equations for coefficients	255
8.5	Numerical results	259
8.6	Conclusions	265
9	CLAMPED CIRCULAR HYBRID PLATE UNDER AXISYMMETRIC LOAD	266
9.1	Introduction	266
9.2	Governing equations	266
9.3	General solution for finite circular plate	268
9.4	Algebraic equations for coefficients	270
9.5	Numerical results	275
9.6	Conclusions	281
10	CONCLUSIONS	282
10.1	Conclusions of the present work	282
10.2	Scope for further work	283
	REFERENCES	284
	BRIEF BIODATA OF THE AUTHOR	292