

**FUZZY SETS VARIANTS FOR HANDLING
UNCERTAINTY IN DECISION MAKING**

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**SCHOOL OF INFORMATION TECHNOLOGY
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**FUZZY SETS VARIANTS FOR HANDLING
UNCERTAINTY IN DECISION MAKING**

by

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Certificate

This is to certify that the thesis titled “Fuzzy Sets Variants for handling Uncertainty in Decision Making” being submitted by Mr. Manish Aggarwal to the Indian Institute of Technology Delhi for the award of the degree of Doctor of Philosophy is a record of bona fide research work carried out by him under our supervision. In our opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The work presented in this thesis has not been submitted elsewhere, either in part or full, for the award of any degree or diploma.

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Abstract

Multi Criteria Decision making (MCDM) in the real world is inevitably associated with the uncertainties on account of vagueness, occurrence, incomplete knowledge and the human factor which is an epitome of subjectivity, attitudes, different priorities, undefined goals and conceptions. MCDM under uncertainty involves imprecise evaluations (by experts) and their aggregation to arrive at an overall decision function for ordering the various alternatives. This thesis deals with the issues of uncertainty evaluation and aggregation in MCDM.

The variants developed in this thesis are generalized intuitionistic fuzzy soft set (GIFSS), information set and rough information set (RIS). GIFSS addresses the issue of distortion of the underlying uncertainty during its representation. The information set gives a quantified representation of the uncertainty in the information source values. RIS is a step ahead and also cater to the imprecision arising from incomplete knowledge. The proposed variants facilitate in the evaluation of vague preferences, objective conditions, imprecise beliefs and any imperfect decision relevant information in the real world.

These fuzzy evaluations form the input for the fuzzy models. Most of the existing fuzzy models are based upon fuzzy rules, and they help to predict the fuzzy output. In the real world, a mix of probabilistic uncertainty and fuzziness is often witnessed. To this end, we develop the probabilistic-fuzzy variants of fuzzy models such as Mamdani-Larsen, Takagi Sugeno and generalized fuzzy models. An approach is developed to compute the probabilistic possibility of the proposed models. This approach is illustrated through the real world applications.

A decision under uncertainty can only be made if the associated evaluations are aggregated in some way. The aggregation of imprecise evaluations may be a complicated process due to the co-existence of different kinds of uncertainties (probabilities, possibilities, and incompleteness), and complex attitudinal character of decision makers. We extend the existing induced aggregation operators to deal with the multi-faceted uncertainty in MCDM. The proposed uncertainty evaluation structures and aggregation operators are applied in real case-studies.

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