

**NONLINEAR CHARACTERISTICS OF PILE FOUNDATIONS  
UNDER HARMONIC VIBRATIONS – FIELD TESTING AND  
ANALYSIS**

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**DEPARTMENT OF CIVIL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY DELHI  
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UNDER HARMONIC VIBRATIONS – FIELD TESTING AND  
ANALYSIS**

*by*

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**Department of Civil Engineering**

*Submitted*

*in fulfilment of the requirements of the degree of Doctor of Philosophy*

**to the**



**INDIAN INSTITUTE OF TECHNOLOGY DELHI**

**APRIL 2018**

*Dedicated to my  
adorable nephew Taj*

## **CERTIFICATE**

This is to certify that the thesis entitled “**Nonlinear Characteristics of Pile Foundations under Harmonic Vibrations - Field Testing and Analysis**”, submitted by **Mr. Sanjit Biswas** to the **Indian Institute of Technology Delhi**, is a record of bonafide research work carried out by him under my supervision and is worthy of consideration for the award of **Doctor of Philosophy** degree. The research reported in this thesis has not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Place: New Delhi

Date:

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## ABSTRACT

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This study includes the determination of soil-pile separation lengths and frequency-amplitude responses of single and group piles under rotating machine induced vertical and coupled i.e. horizontal and rocking vibrations by experimental investigation. Another aspect of the present study is the estimation of boundary zone parameters for single and group piles using the measured responses of the soil-pile systems by numerical analysis. To fulfil these objectives, forced vibration tests were performed in the field on a single pile and a  $2 \times 2$  pile group made of hollow steel piles of length 5.8 m and outer diameter of 0.166 m to determine the frequency-amplitude responses for different eccentric moments under a static load of 15 kN. In addition to that the soil-pile separation lengths of single and group piles are determined by measuring the contact pressure between the pile and soil. The experimental frequency-amplitude responses of the single pile and  $2 \times 2$  pile group are found nonlinear in nature for both vertical and coupled modes of vibration.

The numerical analyses are performed by continuum approach and superposition method to determine the dynamic response of pile foundations using the soil-pile separation lengths obtained from the dynamic pile tests. Comparing the analytical frequency versus amplitude responses with the experimental response curves, the best possible variations of boundary zone soil parameters are determined for different eccentric moments and under vertical and coupled modes of vibration. In this study, the continuum approach is found very efficient for prediction of the nonlinear responses of

single pile as well as pile group with precise inclusion of the boundary zone parameters and soil-pile separation lengths. The variations of stiffness and damping of the soil-pile systems with frequency are presented for all the modes of vibration using the numerical analysis. The pile-soil-pile interaction phenomenon of the  $2 \times 2$  pile group is also investigated by calculating the group efficiency ratios (GER) of the pile group for different eccentric moments and vibration modes. The effect of slenderness ratio of pile, the pile head fixity conditions and the spacing between the piles of pile group on the dynamic response of the pile foundation are demonstrated using the continuum approach analysis. A comparative study is also performed for single pile and  $2 \times 2$  pile group under both vertical and coupled modes of vibration using three different analytical methods: 1. Novak (1974), 2. Novak and El Sharnouby (1983), 3. Novak et al. (1999).

Various design charts are proposed based on the finding of the study, to estimate the soil-pile separation lengths and boundary zone parameters for the single pile and  $2 \times 2$  pile group under vertical and coupled vibrations. The applicability of the proposed design charts is validated for full-scale prototype piles using scaling law. It is found that the proposed design curves can be used to determine the nonlinear frequency-amplitude responses of the full-scale pile foundations under both vertical and coupled vibrations for similar soil-pile conditions. The charts are also validated using experimental data of various literature to check the reliability of the design charts. It is found that the proposed design charts can be used to determine the nonlinear frequency versus amplitude responses of the pile foundations under vertical and coupled vibrations for similar type of soil-pile conditions.

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## सार

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इस अध्ययन में घूमने वाली मशीन के अंतर्गत एकल व समूह पाइल की मृदा-पाइल पृथक्करण की लंबाई एवं आवृत्ति-आयाम प्रतिक्रियाओं का ऊर्ध्वाधर और युग्मित कंपन के अंतर्गत प्रयोगात्मक अन्वेषण किया गया है। वर्तमान अध्ययन का एक अन्य पहलू, सिंगल और ग्रुप पाइल के लिए सीमा के क्षेत्र मानकों का अनुमान है, जो सांख्यिकीय विश्लेषण द्वारा मृदा-पाइल प्रणालियों के मापा प्रतिक्रियाओं का उपयोग करते हैं। इन उद्देश्यों को पूरा करने के लिए, मैदान में खोखले लौह के एक एकल पाइल एवं  $2 \times 2$  - पाइल समूह, जिनकी लंबाई 5.8 मीटर एवं बाहरी व्यास 0.166 मीटर है, विभिन्न विलक्षण क्षणों और 15 kN के एक स्थिर भार के लिए प्राभावित कंपन द्वारा परीक्षण किया गया है और आवृत्ति-आयाम प्रतिक्रिया का निर्धारण किया गया है। एकल और समूह पाइल के मृदा-पाइल पृथक्करण की लंबाई का आंकलन मृदा एवं पाइल के बीच संपर्क दबाव को माप के निर्धारित किया गया है। एकल पाइल और  $2 \times 2$  - पाइल समूह की प्रायोगिक आवृत्ति-आयाम प्रतिक्रिया कंपन के ऊर्ध्वाधर और युग्मित प्रणालियों के लिए प्रकृति में गैर-रेखीय दिखती है।

संख्यात्मक विश्लेषण गतिशील पाइल परीक्षणों से प्राप्त मृदा-पाइल पृथक्करण लंबाई का उपयोग कर पाइल नींव की गतिशील प्रतिक्रिया निर्धारित करने के लिए निरंतर दृष्टिकोण और सुपरपोजी पद्धति द्वारा किया जाता है। प्रायोगिक प्रतिक्रिया घटता के साथ आयाम प्रतिक्रियाओं की तुलना में विश्लेषणात्मक आवृत्ति की तुलना, सीमा क्षेत्र के मृदा के मापदंडों

का सर्वोत्तम संभव रूपांतर विभिन्न विलक्षण क्षणों के लिए और कंपनी के ऊर्ध्वाधर और युग्मित मोड के तहत निर्धारित होते हैं। इस अध्ययन में, निरंतर दृष्टिकोण को गैर-रेखीय प्रतिक्रियाओं की भविष्यवाणी के लिए बहुत ही कुशल पाया गया है। सीमा के मानकों और मृदा-पाइल पृथक्करण लंबाई के सटीक समावेश के साथ एकल पाइल के साथ ही पाइल समूह आवधिकता वाले मृदा-पाइल प्रणालियों की कठोरता और अवमन्दक विविधताएं संख्यात्मक विश्लेषण के उपयोग से कंपनी के सभी तरीकों के लिए प्रस्तुत की जाती हैं।

2 × 2- पाइल समूह के पाइल-मृदा-पाइल घटना की प्रक्रिया को विभिन्न विलक्षण क्षणों और कंपनी मोडों के लिए पाइल समूह के समूह दक्षता अनुपात (जीईआर) की गणना के द्वारा जांच की जाती है। पाइल के धीमे अनुपात का प्रभाव, पाइल के ढक्कन की स्थिति और पाइल समूह के पाइल के बीच की दूरी को पाइल फाउंडेशन की गतिशील प्रतिक्रिया पर प्रदर्शित किया जाता है, सातत्य दृष्टिकोण विश्लेषण का उपयोग करते हुए दिखाया गया है। तीन अलग-अलग विश्लेषणात्मक विधियों का उपयोग करते हुए एकल पाइल और 2 × 2- पाइल समूह के लिए एक तुलनात्मक अध्ययन भी किया जाता है: 1. नोवाक (1974), 2. नोवाक और एल शर्नबी (1983), 3. DYNA5 (1999)।

एकमात्र पाइल के लिए मृदा-पाइल पृथक्करण लंबाई और सीमा क्षेत्र मानकों का अनुमान लगाने के लिए, ऊर्ध्वाधर और युग्मित कंपनों के तहत 2 × 2-पाइल समूह के अध्ययन के आधार पर विभिन्न डिजाइन चार्ट प्रस्तावित किए जाते हैं। प्रस्तावित डिजाइन चार्ट की प्रयोज्यता स्केलिंग सिद्धांत का उपयोग करके पूर्ण-स्केल प्रोटोटाइप पाइल के लिए मान्य है। यह पाया जाता है कि प्रस्तावित डिजाइन वक्र का इस्तेमाल समरूप - मृदा के पाइल परिस्थितियों के लिए दोनों ऊर्ध्वाधर और युग्मित कंपनों के तहत पूर्ण-पैमाने पर पाइल नींव के गैर-रेखीय आवृत्ति-आयाम प्रतिक्रियाओं को निर्धारित करने के लिए किया जा

सकता है। डिजाइन चार्ट की विश्वसनीयता की जांच विभिन्न साहित्यों के प्रयोगात्मक तथ्य का उपयोग करके किया गया है । यह पाया जाता है कि प्रस्तावित डिजाइन चार्ट का उपयोग समान प्रकार की मृदा -पाइल परिस्थितियों के लिए ऊर्ध्वाधर और युग्मित कंपनों के तहत पाइल नींवों के अणुभरण की आवृत्ति के विरुद्ध अमानक आवृत्ति को निर्धारित करने के लिए किया जा सकता है।

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## LIST OF SYMBOLS AND ABBREVIATIONS

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Most of the symbols are defined as they appear in the thesis. Some of the most common symbols which are used repeatedly are listed below.

### Roman symbols

$a_o$	Dimensionless frequency
$a_o^*$	Complex dimensionless frequency
$A$	Cross-sectional area of pile
$B$	Integration constant
$C$	Integration constant
$c$	Cohesion of soil
$c_c^1$	Equivalent viscous damping of single pile in cross direction
$c_c^G$	Equivalent viscous damping of pile group in cross direction
$c_i$	Coefficient of pile internal damping
$c_u^1$	Equivalent viscous damping of single pile in horizontal direction
$c_u^G$	Equivalent viscous damping of pile group in horizontal direction
$c_v$	Equivalent viscous damping
$c_v^1$	Equivalent viscous damping of single pile in vertical direction
$c_v^G$	Equivalent viscous damping of pile group in vertical direction

## List of Symbols and Abbreviations

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$c_{\psi}^1$	Equivalent viscous damping of single pile in rocking direction
$c_{\psi}^G$	Equivalent viscous damping of pile group in rocking direction
$d$	Diameter of pile
$D$	Damping ratio of soil
$D_b$	Damping ratio of soil below pile tip
$D_m$	Damping ratio of inner boundary zone soil
$e$	Eccentricity of rotating masses of oscillator
$E$	Young's modulus of pile
$E_m$	Young's modulus of model pile
$E_p$	Young's modulus of prototype pile
$E_s$	Young's modulus of soil
$f_i$	Dimensionless function
$F$	A coefficient for frequency variation of dynamic interaction factors
$F_i$	Amplitude of axial force
$g$	Acceleration due to gravity
$G$	Shear modulus of soil
$G_b$	Shear modulus of soil below pile tip
$G_m$	Shear modulus of inner boundary zone soil
$h_i$	Length of pile element
$H_i$	Amplitude of horizontal force
$\{H\}$	horizontal forces at the pile heads of a pile group
$i$	Imaginary unit satisfying $i^2 = -1$
$I$	Moment of inertia of pile
$I_m$	Moment of inertia of model pile
$I_p$	Moment of inertia of prototype pile

$k_c^1$	Stiffness of single pile in cross direction
$k_c^G$	Stiffness of pile group in cross direction
$k_{inner}$	Inner boundary zone complex stiffness of soil in horizontal direction
$k_{outer}$	Outer boundary zone complex stiffness of soil in horizontal direction
$k_u$	Equivalent linear complex stiffness of soil in horizontal direction
$k_u^1$	Stiffness of single pile in horizontal direction
$k_u^G$	Stiffness of pile group in horizontal direction
$k_u^i$	Stiffness of an individual pile member for coupled vibration
$k_{ub}$	Complex stiffness of pile base in horizontal direction
$k_{ul}$	Linear complex stiffness of soil in horizontal direction
$k_v$	Equivalent linear complex stiffness of soil in vertical direction
$k_v^1$	Stiffness of single pile in vertical direction
$k_v^G$	Stiffness of pile group in vertical direction
$k_v^i$	Stiffness of an individual pile member for vertical direction
$k_{vb}$	Complex stiffness of pile base in vertical direction
$k_{vl}$	Linear complex stiffness of soil in vertical direction
$k_\psi$	Equivalent linear complex stiffness of soil in rocking direction
$k_\psi^1$	Stiffness of single pile in rocking direction
$k_\psi^G$	Stiffness of pile group in rocking direction
$k_{\psi b}$	Complex stiffness of pile base in rocking direction
$k_{\psi l}$	Linear complex stiffness of soil in rocking direction
$K_0$	Zero order modified Bessel function of second kind
$K_1$	First order modified Bessel function of second kind

## List of Symbols and Abbreviations

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$K_c$	Dynamic stiffness of single pile in cross direction
$K_c^1$	Complex stiffness of single pile in cross direction
$K_c^G$	Complex stiffness of pile group in cross direction
$K_c^n$	Cross stiffness generated due to the coupling between lateral force and moment ( $= K_{u\psi}^n = K_{\psi u}^n$ ) at pile node $n$
$K_v$	Dynamic stiffness of the single pile in vertical direction
$K_v^1$	Complex stiffness of single pile in vertical direction
$K_v^G$	Complex stiffness of pile group in vertical direction
$K_v^n$	Stiffness corresponding to the vertical force at pile node $n$
$K_{vs}$	Static stiffness of single pile in vertical direction
$K_u$	Dynamic stiffness of single pile in horizontal direction
$K_u^1$	Complex stiffness of single pile in horizontal direction
$K_u^G$	Complex stiffness of pile group in horizontal vibration
$K_u^n$	Stiffness corresponding to lateral force at pile node $n$
$K_{us}$	Static stiffness of single pile in horizontal direction
$K_\psi$	Dynamic stiffness of single pile in rocking direction
$K_\psi^1$	Complex stiffness of single pile in rocking direction
$K_\psi^G$	Complex stiffness of pile group in rocking direction
$K_\psi^n$	Stiffness corresponding to moment at pile node $n$
$\{k_b\}$	Stiffness matrix of the soil/element below pile tip
$\{K_{TV}\}$	Total stiffness matrix of pile for vertical vibration
$\{K_{TC}\}$	Total stiffness matrix of pile for coupled vibration

$l$	Length of pile
$m$	Mass of eccentric rotating part in oscillator
$M_i$	Amplitude of the moment in vertical plane
$\{M\}$	Moments at pile heads of a pile group
$n$	Scaling factor for full-scale pile length to model pile length
$N$	SPT value
$N_s$	Static axial force on pile
$p$	Soil resistance in lateral direction
$P$	Dynamic force of oscillator
$\{P\}$	Vertical forces at pile heads of a pile group
$R$	Radius of pile
$R_0$	Radius of pile plus thickness of inner boundary zone soil ( $= R + t_m$ )
$R_b$	Radius of pile tip
$s$	Spacing of piles in pile group
$S_{u1}$	Dimensionless stiffness parameter of equivalent linear soil stiffness in horizontal direction
$S_{u2}$	Dimensionless damping parameter of equivalent linear soil stiffness in horizontal direction
$S_{ub1}$	Dimensionless stiffness parameter of the pile base complex stiffness in horizontal direction
$S_{ub2}$	Dimensionless damping parameter of the pile base complex stiffness in horizontal direction
$S_{ul1}$	Dimensionless stiffness parameter of linear soil stiffness in horizontal direction

## *List of Symbols and Abbreviations*

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$S_{ul2}$	Dimensionless damping parameter of linear soil stiffness in horizontal direction
$S_{v1}$	Dimensionless stiffness parameter of equivalent linear soil stiffness in vertical direction
$S_{v2}$	Dimensionless damping parameter of equivalent linear soil stiffness in vertical direction
$S_{vb1}$	Dimensionless stiffness parameter of the pile base complex stiffness in horizontal direction
$S_{vb2}$	Dimensionless damping parameter of the pile base complex stiffness in vertical direction
$S_{v11}$	Dimensionless stiffness parameter of linear soil stiffness in vertical direction
$S_{v12}$	Dimensionless damping parameter of linear soil stiffness in vertical direction
$S_{\psi1}$	Dimensionless stiffness parameter of equivalent linear soil stiffness in rocking direction
$S_{\psi2}$	Dimensionless damping parameter of equivalent linear soil stiffness in rocking direction
$S_{\psi b1}$	Dimensionless stiffness parameter of the pile base complex stiffness in rocking direction
$S_{\psi b2}$	Dimensionless damping parameter of the pile base complex stiffness in rocking direction
$S_{\psi 11}$	Dimensionless stiffness parameter of linear soil stiffness in rocking direction

$S_{\psi l2}$	Dimensionless damping parameter of linear soil stiffness in rocking direction
$t$	Time
$t_m$	Thickness of boundary zone soil
$u$	Horizontal displacement of pile
$\dot{u}$	Time derivative
$u_n$	Displacement at pile node $n$ in horizontal direction
$v$	Vertical displacement of pile
$\dot{v}$	Time derivative
$v_n$	Displacement at pile node $n$ in vertical direction
$\{v\}$	Vertical displacements at pile heads of a pile group
$U_{1,2,3,4}$	Integration constants
$V_s$	Shear wave velocity of soil
$V_b$	Shear wave velocity soil below pile tip
$W$	Weight of eccentric rotating part in oscillator
$W_s$	Static Load
$y$	Pile deflection in lateral direction
$z$	Depth of pile

**Greek symbols**

$\alpha$	A constant quantifies the dependency of stress level on the system stiffness
$\alpha_1$	Real part of complex dynamic interaction factor
$\alpha_2$	Imaginary part of complex dynamic interaction factor
$\alpha_d$	Complex dynamic interaction factor

## List of Symbols and Abbreviations

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$\alpha_{ij}$	Complex vertical interaction factors of individual piles in a group between piles $i$ and $j$
$\alpha_{ij}^c$	Complex cross interaction factors of individual piles in a group between piles $i$ and $j$ .
$\alpha_{ij}^u$	Complex horizontal interaction factors of individual piles in a group between piles $i$ and $j$ .
$\alpha_{ij}^\psi$	Complex rocking interaction factors of individual piles in a group between piles $i$ and $j$ .
$\alpha_{st}$	Static interaction factor
$[\alpha_H]$	Dynamic group interaction matrix for coupled vibration
$[\alpha_v]$	Dynamic group interaction matrix for vertical vibration
$\beta$	frequency independent soil material damping
$\omega$	Circular frequency
$\psi$	Rocking displacement (rotation) of pile
$\psi_n$	Rotation at pile node $n$
$\varepsilon_{ij}^H$	Complex elements of $inv[\alpha_H]$
$\varepsilon_{ij}^v$	Complex elements of $inv[\alpha_v]$
$\mu$	Mass of pile per unit length
$\nu$	Poisson's ratios for outer zone of soil
$\nu_b$	Poisson's ratio of soil below pile tip
$\nu_m$	Poisson's ratios for inner zone of soil
$\phi$	Complex frequency parameter for vertical vibration
$\varphi$	Friction angle of soil
$\theta$	Eccentric angle between rotating masses of oscillator

$\lambda_{1,2}$  Complex frequency parameters for coupled vibration

**Abbreviations**

AC	Alternating-current
@	At the rate of
BEM	Boundary element method
BH	Borehole
C.G.	Center of gravity
CU	Consolidated undrained
Eq.	Equation
FE	Finite element
FEM	Finite element method
Fig.	Figure
GER	Group efficiency ratio
hp	horsepower
Hz	Hertz
N	Newton
LL	Liquid limit
m	Meter
mm	Millimeter
MPF	Mass participation factor
psi	Pounds per square inch
Pa	Pascal
PL	Plastic limit
Rad	Radian

## *List of Symbols and Abbreviations*

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RPM	Revolutions per minute
s	Second
SPT	Standard penetration test
USCS	Unified soil classification system
°	Degree