STUDY OF FLUID QUEUES AND ITS APPLICATIONS

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STUDY OF FLUID QUEUES AND ITS APPLICATIONS

by

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Dedicated to My Family

Certificate

This is to certify that the thesis entitled ***"Study of Fluid Queues and its Applications"*** submitted by ***Ms. Shruti Kapoor*** to the **Indian Institute of Technology Delhi**, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bona fide research work carried out by her under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

Fluid queues are a specific category of queues where the inflow/outflow of customers or the amount of work in the system changes at a rate dependent on some stochastic process. As in queues, in these models some continuous entity, referred to as fluid, are the individual customers. Flow of the fluid into the reservoir is governed by some stochastic process. The fluid flow, out of the reservoir is controlled by another stochastic process, which can be considered similar to a server, allowing fluid to flow out. The buffer content cannot be negative, so when the buffer content decreases to zero, the buffer content stays zero as long as the constant rate is negative.

The purpose of this thesis is to determine the probability distribution of the buffer content at any given time t for this type of queues. Different methodologies are used to find the solution for various fluid models.

The thesis is divided into six chapters. Chapter 1 is introductory in nature giving a brief description of fluid queues and its applications. A short account of mathematical modeling paradigms used in the thesis like stochastic processes, queues, Markov modulated fluid queues, solution of a fluid queue model and methodologies used to find it are also presented.

Chapters 2 and 3 deal with the fluid queues driven by a birth death process with rational birth and death rates. In Chapter 2, the transient solution is obtained in closed form using the continued fraction approach. We also discuss the concept of an absorbing state and its effect on the solution. However, in Chapter 3, the steady state solution was obtained for the model considered by finding eigen values of the underlying tridiagonal matrix. Two specific models are considered and closed form solutions are obtained for the equilibrium distribution of the buffer occupancy. The solutions obtained in both chapters is verified through numerical illustrations.

Chapter 4 presents the transient behavior of an $M/M/1$ driven fluid queue using simple probability concepts. The transient solution is obtained with the help of recursive functions where the recurrence relations are explained using probability concepts. Further, the transient

solution of the fluid queue driven by an $M/M/1/N$ queue is also obtained. Finally, numerical results are presented for the illustration of the proposed approach.
Chapters 5 and 6 are based on applications of fluid queues.
In Chapter 5, a fluid queue approach to model the quantity or amount of information stored in intermediate nodes and the flow of information in a network based on the IEEE 802.11 protocol is presented. Using the fluid queue model, the transient distribution of the buffer content at any intermediate node is obtained. Certain performance measures relevant to a communication network such as server utilization, average throughput, mean delay and expected buffer content are also obtained at the fluid queue level.
In Chapter 6, a fluid queue is used to find the transient distribution of the amount of charge in a rechargeable battery of finite capacity. The level of charge in the battery is governed by different input and output processes and are dependent on the level of charge in the battery. The considered model has been discussed in [19], and the distribution of the hitting time was found numerically. In this chapter the solution method is based on probabilistic approach and a closed form solution is obtained for the distribution of the level of charge in the battery at any time t. Numerical illutrations are carried out in the end to verify our results.

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ACK	Acknowledgement
ATM	Asynchronous Transfer Model
BDP	Birth Death Process
CDF	Cumulative Distribution Function
CTMC	Continuous Time Markov Chain
DTMC	Discrete Time Markov Chain
EMC	Embedded Markov chain
IEEE	Institute of Electrical and Electronics Engineers Standard Association
LAN	Local Area Network
LCFS	Last Come First Served
MBPS	Mega Bytes Per Second
RSS	Random Service Selection
TCP	Transmission Control Protocol
UPS	Uninterrupted Power Supply
VoIP	Voice over Internet Protocol
WAN	Wide Area Network
WLAN	Wireless Local Area Network