A THESIS ON

FLOW AND HEAT TRANSFER PROBLEMS IN HYDROMAGNETICS

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CERTIFICATE

This is to certify that the thesis entitled "Flow and Heat Transfer Problems in Hydromagnetics", being submitted by Mr. P. Govinda Reddy to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him. Mr. P. Govinda Reddy has worked under my guidance and supervision for two years and two months after taking his Post M.Sc. Diploma (D.I.I.T) in Magneto fluid Mechanics from Indian Institute of Technology, Kharagpur, in 1964. His thesis has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or in full, to any other university or institute for the award of any degree or diploma.

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S Y N O P S I S

During the last decade and half, a new science as well as technology has made itself felt, namely magnetohydrodynamics. Simply, magnetohydrodynamics comes about when an electromagnetic field is coupled with the flow field of an electrically conducting fluid such as mercury or an ionized gas. Originally developed by astrophysicists to study stellar phenomena, the subject is now claimed by engineers as well, because of its many potential technological applications. Among those we might mention the following: Controlled thermonuclear fusion, magnetoaerodynamic drag for re-entry, thrust production for propulsive devices, pumping of liquid metals, high temperature resistant coating, cutting of high temperature alloys, chemical synthesis and liquid metal lubrication. In recent years, the studies relating to flows through porous walled channels have contributed to a large extent to the expanding field of knowledge which is applicable not only to gaseous diffusion technology but also to aerodynamic studies involving the statbilization of boundary layers on wings and diffuser surfaces and transpirational cooling of rocket engines. The study of the motion of visco-elastic fluids in the absence and presence of a magnetic field has applications in many areas. A few examples are the flow of nuclear fuel slurries, flow of plasma, flow of mercury amalgams, handling of biological fluids, flow of blood and lubrication with heavy oils and greases.

The purpose of this thesis is to investigate the flow and heat transfer in hydromagnetic flows in both ordinary Newtonian fluids and visco-elastic fluids, taking into account of the electrical conductivity and porosity of the walls. In view of the complicated nature of system of equations involved, consideration has been restricted to problems of incompressible, electrically conducting liquids through channels of different cross-sections.

The work contained in this thesis has been divided into eight chapters. In chapter I a critical survey of the literature in Magnetohydrodynamics has been carried out.

The steady state solution for the laminar flow of incompressible, viscous and electrically conducting liquid in the presence of transverse magnetic field has been investigated in detail in chapter II. Taking into account the porosity of walls we have obtained the velocity profiles, the profiles for the induced magnetic and electric fields. Consideration has also been given to the rectangular channels in the following two cases : (a) when the total current is zero and (b) when the total current is not zero. It is found that the general solution contains two unknown constants. By properly choosing these constants we have shown that the same general solution can be made to fit with the solution for all two-dimnesional channels (rectangular and annular) whose geometry approaches in the limit to that of the one dimensional channel. It is found that near the suction wall the viscous forces are much more pronounced than at the injection wall.

Chapter III deals with the problem of investigating the influence of transverse magnetic field on the laminar radial flow of an incompressible fluid between two closely spaced parallel plates. The governing equations and boundary conditions are established and a solution is carried out by perturbing the creeping flow solution and an asymptotic expansion is made out. The extent of the effect of the magnetic field on the flow is determined by the magnitude of the Hartmann number and the numerical results for the pressure distribution are presented. Entrance effects are being neglected. It has been concluded that the pressure increases with the increase of the magnetic field.

Two-dimensional steady-state incompressible hydromagnetic flow through a rectangular channel in the presence of uniform transverse magnetic field has been investigated in chapter IV when the fluid is being withdrawn through both walls of the channel at different rates say v_1 and v_2. Perturbation solutions have been obtained for small values of the cross Reynolds' numbers for the velocity components and pressure distribution. It has been noticed that the pressure drop in the major flow direction increases with the increase of the Hartmann number, M, which determines the intensity of the magnetic field and decreases with the increase of the suction parameters R_2 and α_2. Some more interesting results have been recorded for the skin-friction near the walls and streamlines have been drawn.

In chapter V we study the effect of the magnetic field on the flow of viscous, incompressible and electrically conducting fluid in the presence of a transverse magnetic field through long rectangular ducts under the condition that the axial pressure gradient is an arbitrary function of time. The solution for an impulsive pressure gradient is first derived by transforming the governing equation of motion to a homegeneous one along with appropriate changes in initial conditions. The cases of other time dependent pressure gradients, such as (i) constant pressure gradient (ii) harmonically oscillating pressure gradient and (iii) a purely harmonic motion superimposed on a steady flow, are discussed with the help of the convolution integral or Duhamel's integral. Numerical values of the important flow characteristics such as transient velocity distribution, mass flow rate, sectional average velocity, shear stress and energy dissipation are presented for different values of the Hartmann number and the aspect ratio of the duct cross-sectional area. The following observations can be drawn from the graphs and tables presented. In the case of constant pressure gradient, the effect of the magnetic field is to flatten the velocity profiles and to shorten the flow development time. It increases the frictional factor and decreases the mass flow rate. The velocity near the duct axis and the pressure force has a 90° phase shift in the case of harmonically oscillating pressure gradient. The presence of the magnetic field affects considerably the velocity profiles and there is a marked difference in the case of fast oscillations than in the case of small values of ψ, the frequency parameter. The maximum velocity in the case of large ψ, does not occur on the axis of the duct but it falls near the wall. In the case of constant pressure gradient superimposed on a harmonically oscillating pressure gradient, the value of the coefficient of excess work, $G(\psi)$, decreases as ψ increases whatever may be the values of ϕ, the aspect ratio of the duct cross-section, and M, the Hartmann number, which means the faster the oscillations the less extra energy being dissipated. $G(\phi)$ increases as magnetic field strength increases.

In chapter VI the analytical solution to the equation of motion is given for the steady laminar flow of a uniformly incompressible visco-elastic fluid between two parallel plates when the plates are electrically conducting. The constitutive equation for visco-elastic fluid employed here is one in which the extra stress p^i_j is related to the rate of strain tensor d^i_j in the form

$$p^i_j + \gamma \bar{p}^i_j = 2\mu d^i_j + 4\mu_c d^i_\alpha d^\alpha_j, \qquad (i)$$

where γ, the elastic constant having dimensions of time, μ is the coefficient of viscosity, μ_c is the coefficient of cross-viscosity and $\bar{p}^i_j$ represents the rate of stress which following truesdell or Oldroyd for incompressible fluids, is given as

$$\bar{p}^{ij} = \frac{\partial p^{ij}}{\partial t} + p^{ij}_{,k} v^k - p^{ik} v^j_{,k} - p^{kj} v^i_{,k} + p^{ij} v^k_{,k}. \qquad (ii)$$

The fluid is under the influence of a constant pressure gradient and is subjected to a uniform magnetic field perpendicular to the direction of motion. The extent of the effect of wall electrical conductance has been determined by the parameters ϕ_l and ϕ_u which are the wall electrical conductance ratios of the lower and upper walls. Flow rates, velocity profiles, distribution of electric current and the induced magnetic field profiles for various values of ϕ_l, ϕ_u, M and R_c, the visco-elastic parameter, are presented and compared with those corresponding to Newtonain fluid.

Chapter VII contains the investigation of the influence of wall conductance on the combined forced and free convection of an electrically conducting fluid flowing inside an open ended vertical channel formed by two identical parallel walls in the presence of a uniform transverse magnetic field. The surface temperatures of the walls have been considered varying linearly along the vertical distance. Exact solutions have been obtained for velocity, temperature and magnetic field distributions neglecting the viscous dissipation. The effect of constant heat sources and sinks has also been investigated. Numerical calculations have been carried out on the flow, heat and magnetic characteristics for various combinations of the non-dimensional parameters involved in the problem. We notice that the effects of the electrical conductivity of the walls on the flow characteristics are considerable. We see that for a given value of $(\phi_1+\phi_2)$, the sum of the wall conductivity ratios, increase in M results in flattening the velocity profiles while sharpening the temperature distributions. When vertical temperature gradient is negative a critical Raleigh number is found at which instability appears and increasing magnetic field is shown to have stabilizing effect.

The effect of wall conductance on a laminar flow of a uniform conducting incompressible flow is established in chapter VIII between two parallel walls under a uniform magnetic field perpendicular to the walls; the temperature distribution within the fluid and the local Nusselt number are theoretically investigated for the thermal entrance region. Boundary conditions are for both the cases of prescribed uniform heat flux at wall and that of uniform wall temperature. Joule heating is taken into account but heat produced by viscous force is neglected. The use of orthogonal functions to solve the energy equation leads to an eigenvalues problem of Sturm-Liouville type differential equation which has been solved by Ritz method. Numerical calculations of the results are done.

The numerical computation of the results presented in this thesis has been carried out on the electronic Digital Computer, IBM 1620 installed at Indian Agricultural Research Statistics Institute, New Delhi.

C O N T E N T S