FLOW AND CONVEX-CONCAVE RATIO PROBLEMS : SOME ASPECTS OF OPTIMIZATION

By

ANJU GUPTA

Department of Mathematics

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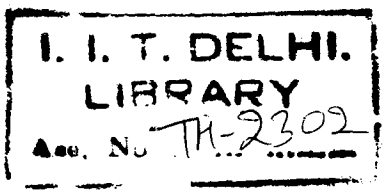

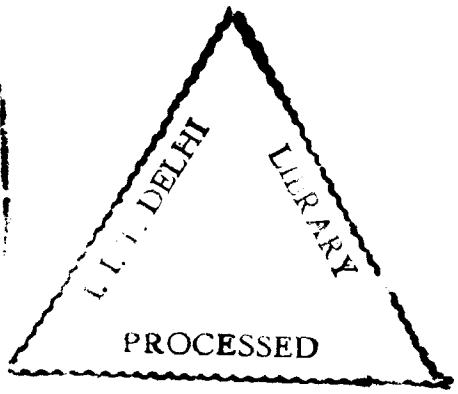

CERTIFICATE

This is to certify that the thesis entitled

FLOW AND CONVEX-CONCAVE RATIO PROBLEMS : SOME ASPECTS OF OPTIMIZATION

which is being submitted by ***Ms. Anju Gupta*** *for the award of the degree of* ***DOCTOR OF PHILOSOPHY in MATHEMATICS*** *to the* ***INDIAN INSTITUTE OF TECHNOLOGY, DELHI,*** *is a bonafide record of research work done under my guidance and supervision.*

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

M C Puri

(M.C. PURI)

Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi - 110016
INDIA.

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(ANJU GUPTA)

Department of Mathematics,
Indian Institute of Technology,
Hauz Khas, New Delhi - 110016,
INDIA.

ABSTRACT

The thesis entitled "Flow and Convex-Concave Ratio Problems: Some Aspects of Optimization" deals with certain aspects of optimization arising in the flow and the convex-concave ratio problems. The subject matter has been arranged in the thesis in five chapters.

Chapter I consisting of two sections is an Introductory chapter. In the Section I, a brief survey of the work carried out by various authors in the relevant fields is given, while the Section II gives a brief summary of the work carried out in the subsequent four chapters of the thesis.

Chapter II consisting of two sections deals with the flow problems wherein the flow of goods and services are carried out in a transportation network, from one node to the other. The sufficient conditions for the existence of a paradox in the transportation problem with mixed constraints have been evolved in the Section I. Section II discusses the 'paradoxical solutions' in the minimal cost network flow problems with bounded variables.

Another important class of transportation problems linked with the supply of material from different sources to various warehouses with the constraint that each warehouse will draw its total supply from only one source-commonly termed as Bulk Transportation Problem, is dealt with in Chapter III. Algorithm for ranking techniques has been demonstrated for solving the quadratic bulk transportation problem in Section II. Section III develops the methodology to solve the fractional bulk transportation problem.

Chapter IV is devoted to studying another facet of the bulk transportation problem that involves simultaneous study of the two conflicting criteria. In Section I, an algorithm for solving the general bicriteria bulk transportation problem is developed, while in Section II a technique of trading-off cost against time in the bulk transportation problem is developed.

Chapter V, the last chapter, discusses a non-concave fractional programming problem of convex-concave type. It consists of two sections. In Section I, a non-concave fractional objective function is optimized over the set of linear constraints whereas in Section II different algorithm is developed for optimizing the same problem under the transportation constraints.

Numerical examples are given to illustrate the various techniques discussed in the thesis.

The work in the thesis is presented as follows :

References to paper/books mentioned in the thesis are given at the end and number in the brackets [ ] refer to these reference.

The numbering of various results is such that first number represents the chapter, second the section of that chapter and third the serial number of the result in that section.

Problems are numbered as P.a.b.c. which refers to the problem number c of the section b of the chapter a. Equations/inequations are represented by the numbers such that the first number represents the chapter and the second the serial number of the equation/inequation.

LIST OF PUBLICATIONS

1. Paradoxical Situations in Transportation Problems, Cahiers du Centre d' Etudes de Recherche Operationnelle, vol. 34, no. 1, 1992, pp. 37-49.

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7. Maximizing Pseudoconvex Transportation Problem : A Special Type, accepted for publication, to appear in OR-Spektrum in 1995.

8. An Extreme Point Ranking Technique for a class of Pseudoconvex Maximization Problem, submitted for publication.

9. Standard Transportation Problem : A special Type, Proceedings of First Annual Conference of Indian Society of Industrial and Applied Mathematics, 1992, pp. 354-357.

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