# NETWORK AND PRICING MODULARITY IN ELECTRICITY MARKETS

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by

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*I dedicate this thesis to my loving mother and father for all their love and support and putting me through the best education possible. I appreciate their sacrifices and I wouldn't have been able to get to this stage without them.*

# Certificate

This is to certify that the dissertation entitled '**Network and Pricing Modularity in Electricity Markets**', being submitted by **Mr. Deep Kiran** for the award of the degree of **Doctor of Philosophy** is a record of bonafide research work carried out by him in the Department of Electrical Engineering at Indian Institute of Technology Delhi, New Delhi.

**Mr. Deep Kiran** has worked under our supervision and has fulfilled the requirements for the submission of this dissertation, which to our knowledge has reached the requisite standard. The results obtained here have not been submitted to any other University or Institute for the award of any degree.

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# Abstract

The research work presented in this dissertation discusses various complex issues associated with network and pricing modularity in electricity markets. The existing approaches in this respect need significant revision and, thus the work in this dissertation primarily aims at developing new efficient approaches to serve this need. The independent system operator (ISO) is responsible for the network and pricing modularity in electricity markets. So long as network modularity is concerned, the ISO carries out the same well before the day of operation with the motive of economical operation. This market could not be perfectly economical because of many factors. Therefore, after the completion of the day of operation, the ISO takes some additional steps amounting to revenue reconcilement. Some of these steps can be categorized under pricing modularity present in electricity markets.

With the introduction of competition and associated modes of transaction, network segmentation with reference to congestion became a necessity. This dissertation presents a methodology to solve network segmentation problem in a wholesale competition environment so that a set of appropriate bid area is formed. A two-step process is presented. The first step involves optimization based clustering of nodes with the closeness of locational marginal prices (LMP) as a decision metric. The second step involves fine tuning of clusters from the first step by bus migration process. It is observed that the performance index improves under certain cases of bus migration. However, a pre-requisite in the second step is a threshold input which defines the number of zones that can be formed on a network. In this dissertation, the concept of linear bottleneck games (LBG) is introduced in the second step for the formation of congestion-free bidding zones that includes nodes with the similar contribution to the congested lines. This second step identifies the optimal zonal combination and crucial nodes by solving

a primal-dual linear programming (PDLP) model. It is observed that the buses with similar LMPs lie in the same zone, subject to their connectivity.

Another problem in power system for node aggregation is to form multiple zones for the allocation of transmission usage prices which depends on a set threshold, connectivity and type of pricing mechanism. In this dissertation, a novel concept to club the nodes into zones that are closely connected and experience similar usage prices, for few of the popular transmission pricing mechanisms is proposed. Also, a quality index as a function of edge weights, which uses a concept of *edge betweenness*, is proposed to decide on the optimal number of zones. It is observed that the buses with similar transmission usage prices with respect to their mega-watt (MW) quantum lie in the same zone.

To this end, one of the important contributions in this dissertation is the new optimal power flow (OPF) model. The direct current (DC)-OPF and alternating current (AC)-OPF models have been extensively used for solving OPFs. There is another OPF model that lies in between these models, i.e., known as loss compensated (LC)-DC-OPF, which incorporates the effects of network losses in DC-OPF model. This dissertation presents the LC-DC-OPF model which is based on quadratically constrained quadratic programming (QCQP). Also, it presents an approach to decompose the market-settlement surplus (MS) into network losses and congestion related components in LC-DC-OPF model using its Karush-Kuhn-Tucker (KKT) conditions. The decomposition of MS may further establish a proper mechanism for hedging against network losses and congestion. When the components of MS are reproduced using the components of LMP, it does not sum up to MS. That precludes mapping of the LMP components one-on-one with those of MS components, making the comparison implausible. This dissertation also proposes an exact method to determine the components of MS exist in the present LMP market. It highlights the reason for the anomaly that exists in the current practice because of which, in literature, it is shown that the network is split into multiple regions to get multiple energy components of LMP.

Within an AC-OPF market clearing framework, MS is inevitable due to network losses, congestion and reactive power, therefore, its decomposition into its constituents is a challenge. Therefore, this dissertation proposes a solution for decomposing MS into its constituents, i.e., network losses, congestion and reactive support services, by deriving analytical expressions for each of the components. An optimization problem, named as *pseudo*-IV-OPF (P-IV-OPF), is set-up around the existing AC-OPF solution to derive expressions. The central idea involved in current and voltage OPF (IV-OPF) that the current flow at sending end is exactly equal and opposite to that at receiving end, is exploited to derive the elements of MS by using KKT conditions of optimality. It is important to note that a converged OPF solution, obtained from the standard AC-OPF model used by the utility, is input to the proposed model. The advantage of the proposed derivation is that the nodal pricing definitions do not change. Thus, the proposed model can be incorporated in the electricity markets by the ISO with ease.

In the LC-DC-OPF markets, the reference bus plays a crucial role in the decomposition of LMP components for the correct valuation and settlement of financial instruments in these markets. Therefore, this dissertation proposes a simple non-marginal method for calculating reference bus independent components of LMP in the LC-DC-OPF models used in current practice. The reference bus independency could be useful in establishing different financial instruments in the electricity market.

In AC formulations, reactive power plays a crucial role and researchers have faced challenges in incorporating it into the standard market design (SMD). In this dissertation, an exact decomposition of LMP is proposed in AC formulation to provide a complete pricing SMD. The proposed work uses the concept of decomposition of MS in the AC-OPF SMD and extends it to derive the components of LMP, i.e., real power, network losses, congestion and reactive support services, using the concept of circuit and graph theory. The proposed framework unveil interesting features of the LMP components that were undercover until now.

The proposed methodologies have been extensively tested on multiple test systems and the detailed discussions on results from various case studies have been presented. The results of these test systems illustrate the effectiveness and the efficacy of the proposed approaches and provide insights into the nature of the problem.

# सार

इस निबंध में प्रस्तुत अनुसंधान कार्य बिजली मार्केट्स में नेटवर्क और मूल्य निर्धारण प्रतिरूपकता के साथ जुड़े विभिन्न जटिल मुद्दों पर चर्चा करता है। इस संबंध में मौजूदा दृष्टिकोणों में महत्वपूर्ण संशोधन की आवश्यकता है और इस प्रकार इस निबंध में काम मुख्य रूप से इस आवश्यकता को पूरा करने के लिए नए कुशल तरीकों को विकसित करने का लक्ष्य है। बिजली के मार्किट में स्वतंत्र सिस्टम ऑपरेटर (आईएसओ) नेटवर्क और मूल्य निर्धारण प्रतिरूपकता के लिए जिम्मेदार है। जहा तक नेटवर्क प्रतिरूपकता का सम्बन्ध है, आईएसओ आर्थिक संचालन के मकसद से संचालन के वास्तविक दिन से पहले ही इन जिम्मेदारियों को निभाता है। कई कारकों के कारण यह मार्किट पूरी तरह से किफायती नहीं हो सकता। इसलिए, संचालन के दिन के पूरा होने के बाद, आईएसओ कुछ अतिरिक्त कदम उठाता है जो राजस्व समाधान के लिए होता है। इन कदमों में से कुछ बिजली के मार्किट में मूल्य निर्धारण प्रतिरूपकता के अंतर्गत वर्गीकृत किया जा सकता है।

प्रतियोगिता और लेनदेन के सम्बद्ध की शुरुआत के साथ, संकुलन के सन्दर्भ में नेटवर्क विभाजन एक आवश्यकता बन गया है। यह निबंध एक थोक प्रतिस्पर्धा माहौल में नेटवर्क विभाजन समस्या को हल करने के लिए एक पद्धति प्रस्तुत करता है ताकि उचित बोली क्षेत्र का एक समूह बन जाए। इसकेलिए दो-चरण की प्रक्रिया प्रस्तुत की गई है। पहले चरण में स्थानिक सीमांत कीमतों (एलएमपी) के निकटतम निर्णय मीट्रिक के रूप में नोड्स के अनुकूलन आधारित क्लस्टरिंग शामिल है। दूसरे चरण में बस प्रवासन प्रक्रिया के पहले चरण से क्लस्टर की ठीक ट्यूनिंग शामिल है। यह देखा गया है कि प्रदर्शन सूचकांक बस प्रवासन के कुछ मामलों में सुधार करता है। हालांकि, दूसरे चरण में एक पूर्व-आवश्यकता एक दहलीज इनपुट है जो एक नेटवर्क पर बनाई जा सकने वाले क्षेत्र की संख्या को परिभाषित करता है। इस निबंध में, संकुलन-मुक्त बिडिंग क्षेत्र

के गठन के लिए लीनियर बोटलनेक गेम्स (एलबीजी) की अवधारणा को दूसरे चरण में पेश किया गया है जिसमें नोड्स को संकुलित लाइनों में समान योगदान के आधार पर विभाजित किया गया है। यह दूसरा चरण एक प्राइमल-ड्यूल लीनियर प्रोग्रामिंग (पीडीएलपी) मॉडल को हल करके इष्टतम क्षेत्रीय संयोजन और महत्वपूर्ण नोड्स की पहचान करता है। यह देखा गया है कि इसी तरह की एलएमपी वाली बसें उसी क्षेत्र में हैं, जो उनकी कनेक्टिविटी के अधीन हैं।

नोड एकत्रीकरण के लिए बिजली व्यवस्था में एक और समस्या ट्रांसमिशन उपयोग की कीमतों के आवंटन के लिए एक से अधिक क्षेत्रो का निर्माण करती है जो एक निर्धारित दहलीज, कनेक्टिविटी और मूल्य निर्धारण तंत्र के प्रकार पर निर्भर करती है। इस निबंध में, एक नई अवधारणा प्रस्तुत की गई है जो नोड्स के निकटता से जुड़े होने, समान उपयोग मूल्यों का अनुभव करने और कुछ लोकप्रिय ट्रांसमिशन मूल्य निर्धारण तंत्र के आधार पर क्षेत्रो में समूहीकरण करता है। साथ ही, एज बेटविण्णेस की अवधारणा का उपयोग करने वाले फ़ंक्शन के रूप में एक गुणवत्ता सूचकांक जो क्षेत्र के इष्टतम नंबर पर निर्णय लेने का प्रस्ताव है। यह देखा गया है कि जो नोड्स अपने मेगा-वाट क्वांटम के संबंध में समान ट्रांसमिशन उपयोग की मूल्यांकन किया गया है वे सारे बसेस एक ही क्षेत्र में स्थित है अपने कनेक्टिविटी के आधार पर।

इस अंत में, इस निबंध में महत्वपूर्ण योगदान में से एक नया ऑप्टीमल पावर फ्लो (ओपीएफ) मॉडल है। डायरेक्ट करंट (डीसी)-ओपीएफ और अल्टेरनेटिंग करंट (एसी)-ओपीएफ मॉडलों को बड़े पैमाने पर ओपीएफ सुलझाने के लिए इस्तेमाल किया गया है। एक और ओपीएफ मॉडल है जो इन मॉडलों के बीच स्थित है जिसे लोस्स कम्पेंसेटेड (एलसी)-डीसी-ओपीएफ के रूप में जाना जाता है, जो डीसी-ओपीएफ मॉडल में नेटवर्क लॉसेस के प्रभाव को शामिल करता है। यह निबंध एलसी-डीसी-ओपीएफ मॉडल प्रस्तुत करता है जो क्वाड्रटिकालय कन्स्ट्रैनेड क्वाड्रटिक प्रोग्रामिंग (कककप) पर आधारित है। इसके अलावा, यह एलसी-डीसी-ओपीएफ मॉडल अपने करूष-कुहन-टकर (केकेटी) शर्तों का उपयोग करके अपने मार्किट-सेटलमेंट सरप्लस (एमएस) को नेटवर्क लॉसेस और संकुलन

संबंधित घटकों में विघटित करने के लिए एक दृष्टिकोण प्रस्तुत करता है। एमएस का अपघटन नेटवर्क लॉसेस और संकुलन के खिलाफ हेजिंग के लिए एक उचित तंत्र स्थापित कर सकता है। जब एमएस के घटकों को एलएमपी के घटकों का उपयोग करके पुन: प्रजनित किया जाता है, तो यह एमएस से सम्बद्ध नहीं होता है। यह एमएस घटकों के साथ एक-एक कर एलएमपी घटकों की मैपिंग को रोकता है, जो तुलना में असंभव है। वर्तमान में एलएमपी व्यापार में एमएस के घटकों को निर्धारित करने के लिए यह शोध प्रबंध एक सटीक पद्धति भी प्रस्तावित करता है। यह मौजूदा अभ्यास में मौजूद असंगति के कारणों को उजागर करता है जिसकी वजह से साहित्य में यह दिखाया गया है कि एलएमपी के कई ऊर्जा घटकों को प्राप्त करने के लिए नेटवर्क को कई क्षेत्रों में विभाजित किया जा सकता है।

एसी-ओपीएफ़ मार्केट क्लियरिंग फ्रेमवर्क के भीतर, नेटवर्क लॉसेस, संकुलन और प्रतिक्रियाशील शक्ति के कारण एमएस अपरिहार्य है, इसलिए, इसके घटकों को निर्धारित करना एक चुनौती है। इसलिए, इस शोध में प्रत्येक घटकों के लिए विश्लेषणात्मक अभिव्यक्तियां निकालने के द्वारा एमएस को अपने घटकों, यानी नेटवर्क लॉसेस, संकुलन और प्रतिक्रियाशील समर्थन सेवाओं में विघटित करने का समाधान प्रस्तावित करता है। सूडो-आईवी-ओपीएफ (पी-आईवी-ओपीएफ) नामक एक ऑप्टिमाइज़ेशन मॉडल का सेट-उप किया गया है जिसका सलूशन मौजूदा एसी-ओपीएफ सलूशन के आसपास हो ताकि अभिव्यक्तियां को प्राप्त किया जा सके। करंट और वोल्टेज ओपीएफ (आईवी-ओपीएफ) में शामिल केंद्रीय विचार यह है कि करंट प्रवाह भेजने वाली बस और प्राप्त करने वाली बस पर बराबर और विपरीत रहती है, एमएस के तत्वों को अनुकूलतमता की केकेटी शर्तों का उपयोग करके प्राप्त किया गया है। यह ध्यान रखना जरूरी है कि उपयोगिता द्वारा उपयोग किए गए मानक एसी-ओपीएफ़ मॉडल से प्राप्त समेकित ओपीएफ सलूशन प्रस्तावित मॉडल के लिए इनपुट है। प्रस्तावित व्युत्पत्ति का लाभ यह है कि नोडल मूल्य निर्धारण की परिभाषाएं बदलती नहीं

हैं। इस प्रकार, प्रस्तावित मॉडल आईएसओ द्वारा आसानी से बिजली मार्किट में शामिल किया जा सकता है।

एलसी-डीसी-ओपीएफ़ मार्केट्स में, संदर्भ बस, एलएमपी घटकों के अपघटन में महत्वपूर्ण मूल्यांकन और वित्तीय साधनों के निपटान के लिए महत्वपूर्ण भूमिका निभाता है। इसलिए, इस शोध में मौजूदा अभ्यास में इस्तेमाल होने वाले एलसी-डीसी-ओपीएफ़ मॉडल में एलएमपी के संदर्भ बस स्वतंत्र घटकों की गणना के लिए एक सरल गैर-सीमांत विधि का प्रस्ताव किया गया है। संदर्भ बस स्वतंत्रता बिजली मार्किट में विभिन्न वित्तीय उपकरणों की स्थापना में उपयोगी हो सकता है।

एसी फॉर्मूलेशन में, प्रतिक्रियाशील शक्ति एक महत्वपूर्ण भूमिका निभाती है और शोधकर्ताओं ने इसे स्टैण्डर्ड मार्किट डिज़ाइन (एसएमडी) में शामिल करने में चुनौतियों का सामना किया है। इस निबंध में, एसी मॉडल के एलएमपी का एक सही अपघटन तैयार करने का प्रस्ताव किया गया है ताकि पूर्ण मूल्य निर्धारण एसएमडी प्रदान किया जा सके। प्रस्तावित कार्य एसी-ओपीएफ एसएमडी में एमएस के अपघटन की अवधारणा का उपयोग करता है और सर्किट और ग्राफ सिद्धांत की अवधारणा का उपयोग करते हुए एलएमपी को वास्तविक शक्ति, नेटवर्क लॉसेस, संकलन और प्रतिक्रियाशील समर्थन सेवाओं के घटकों को प्राप्त करने के लिए विस्तारित करता है। प्रस्तावित ढांचा अब तक एलएमपी घटकों की रोचक विशेषताओं का खुलासा करता है जो अब तक गुप्त थे।

प्रस्तावित तरीकों को कई परीक्षण प्रणालियों पर बड़े पैमाने पर परीक्षण किया गया है और विभिन्न केस स्टडीज के परिणामों पर विस्तृत चर्चाएं प्रस्तुत की गई हैं। इन परीक्षण प्रणालियों के परिणाम प्रस्तावित दृष्टिकोणों की प्रभावशीलता और प्रभावकारिता को समझाते हैं और समस्या की प्रकृति में अंतर्दृष्टि प्रदान करते हैं।

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# Nomenclature

The chapter wise symbols are given in this nomenclature. Some symbols are defined as and when required. Also, some symbols are defined as per standard convention.

**Symbols used in Chapter 2**

| | |
|---|---|
| $\alpha, \beta, \eta, \varepsilon, \delta$ | Weights of metrics in aggregate fitness function. |
| $\gamma_{z_\kappa}$ | The zonal probability for a zone formation $\forall z_\kappa \in N$. |
| $\lambda$, $\mu$, $\nu$ | Lagrange multipliers. |
| $\sigma$ | Scaling factor. |
| $\underline{y}$, $\overline{y}$ | Lower and upper bounds. |
| $DF_v^e$ | Distribution factor of each $v$ on $e$. |
| $E$ | Number of edges. |
| $e$ | Edge. |
| $f_t$ | Cost function of each $t$. |
| $g_v$ | Nodal power balance equations of $v$. |
| $h_e$ | Flow constraints of $e$. |
| $I$ | Number of zones from which $z_\kappa$ imports power. |
| $K$ | Number of nodes connected to elastic entities. |

| | |
|---|---|
| $k$ | Zonal network, $k \in \{1, \ldots, K\}$. |
| $Q_{z_\kappa}$ | Elastic entity identifier, (1, if any $t \exists z_\kappa$ and 0, otherwise). |
| $T$ | Number of elastic entities. |
| $t$ | Elastic entity. |
| $V$ | Number of nodes. |
| $v, p, q, r$ | Node. |
| $x_v$ | The contribution of each node in $z_\kappa, \forall v \in z_\kappa$. |
| $y_t$ | Control parameter vector that corresponds to the injections by $t$ (positive for suppliers and negative for consumers). |
| $z_\kappa$ | Nodal combination, $\kappa \in \{1, \ldots, k\}$ which is a vector of $\{0,1\}^V$ with $v^{th}$ element of $\kappa^{th}$ zone is equal to *one*, if $v \in z_\kappa$ and *zero*, otherwise. |

### Symbols used in Chapter 3

| | |
|---|---|
| $\gamma_e^{ij}$ | The generation shift factors on line $e$. |
| $\llcorner\sigma_{ij}$ | The lower triangular matrix of $\sigma_{ij}$. |
| $\omega_i$ | Nodal transmission usage price at net withdrawal node $i$. |
| $\sigma_{ij}$ | The edge weight of an edge connected between net withdrawal node $i$ and $j$. |
| $d_s$ | The sum of the degrees of the nodes in zone $s$. |
| $E$ | The number of edges. |
| $F_e$ | The power flow of line $e$. |
| $F_e^i$ | The power flow in line $e$ when nodal injection of net withdrawal node $i$. |

| | |
|---|---|
| $GD_{ij}$ | Share between net injection ($i$) and withdrawal ($j$) nodes found by proportionality principle. |
| $l_s$ | The number of edges between nodes within zone $s$. |
| $M$ | A large value. |
| $N$ | The number of zones. |
| $P_i$ | The original nodal withdrawal at net withdrawal node $i$. |
| $P_{D_j}$ | Real power withdrawal at $j^{th}$ net withdrawal node exists in set of net withdrawal nodes ($D$). |
| $P_{G_i}$ | Real power injection at $i^{th}$ net injection node exists in set of net injection nodes ($G$). |
| $Part_{ie}$ | The proportion of transmission usage charges $\omega_i$ at net withdrawal node $i$. |
| $PD$ | Total real power demand of the system. |
| $Q_E$ | A quality index in terms of edges. |
| $Q_N$ | A quality index in terms of nodes. |
| $T_s$ | Weighted average price of each zone $s$. |
| $V$ | The number of net withdrawal nodes. |

**Symbols used in Chapter 4**

| | |
|---|---|
| $[\bullet]$ | Lagrange multipliers corresponding to each equation. |
| $\delta_i$ | Voltage angle at bus $i$. |
| $\lambda_i$ | LMP at bus $i$. |

| | |
|---|---|
| $\lambda_i^e$ | Energy component of LMP at bus $i$. |
| $\lambda_i^{cong}$ | Congestion component of LMP at bus $i$. |
| $\lambda_i^{loss}$ | Loss component of LMP at bus $i$. |
| $\omega$ | An offset for transmission losses. |
| $\underline{(\bullet)}$ and $\overline{(\bullet)}$ | Lower and upper bounds of variable $(\bullet)$. |
| $D_i$ | Real power demand at bus $i$. |
| $DF$ | A vector of loss distribution factor representing the distribution of losses. |
| $f$ | The cost function. |
| $G_i$ | Real power generator at bus $i$. |
| $L$ | A vector of loss. |
| $LF$ | A vector of loss factor. |
| $P_{ij}$ | Real power flow in line $ij$. |
| $P_{ij}^l$ | Real power loss in line $ij$. |
| $P_{ij}^t$ | Virtual variable corresponding to the modulus of $P_{ij}$. |
| $T$ | Power transfer distribution factor. |
| $x_{ij}$ | Reactance of line $ij$. |

**Symbols used in Chapter 5**

| | |
|---|---|
| $(\bullet)^-$,$(\bullet)^+$ | Dual variable corresponding to lower and upper limit of the variable $(\bullet)$. |
| $(\bullet)^*$ | Complex conjugate of $(\bullet)$. |
| $(\bullet)^R$,$(\bullet)^I$ | Lagrange multiplier corresponding to real and imaginary part of $(\bullet)$. |

| | |
|---|---|
| $\$_{\Im(loss)}$ | Surplus corresponds to reactive power loss. |
| $\$_{\Re(loss)}$ | Surplus corresponds to real power loss. |
| $\$_{cong}$ | Surplus corresponds to congestion. |
| $\$_{gen}$ | Generator payment. |
| $\$_{lc}$ | Surplus corresponds to reactive support from line charging. |
| $\$_{load}$ | Load payment. |
| $\$_{loss}$ | Surplus corresponds to network losses. |
| $\$_{Q}$ | Surplus corresponds to reactive support from generators and loads. |
| $\$_{sh}$ | Surplus corresponds to reactive support from shunts. |
| $\delta_i$ | Dual variable corresponding to voltage limit at $i^{th}$ bus. |
| $\gamma_i$ | Dual variable corresponding to nodal power balance equations at $i^{th}$ bus. |
| $\Im(G)^{NREG}$ | Set of non-regulating generators and non-generator buses. |
| $\jmath$ | Imaginary number. |
| $\lambda_i$ | Dual variable corresponding to nodal current balance equations at $i^{th}$ bus. |
| $\mu_i$ | Dual variable corresponding to power injection by generator at $i^{th}$ bus. |
| $\nu_{ij}$ | Dual variable corresponding to power flow in line connected between $i^{th}$ and $j^{th}$ bus. |
| $\omega_{ij}$ | Dual variable corresponding to current flow in line connected between $i^{th}$ and $j^{th}$ bus. |

| | |
|---|---|
| $\phi_i$ | Dual variable corresponding to current injection by shunt at $i^{th}$ bus. |
| $\pi_{ij}$ | Dual variable corresponding to effective power flow limit in line connected between $i^{th}$ and $j^{th}$ bus. |
| $\Re(\bullet)$,$\Im(\bullet)$ | Real and imaginary component of the variable $(\bullet)$. |
| $\Re(G)^M$ | Set of marginal generators. |
| $\theta_{ij}$ | Dual variable corresponding to power flow limit in line connected between $i^{th}$ and $j^{th}$ bus. |
| $\underline{(\bullet)}$,$\overline{(\bullet)}$ | Lower and upper limit of the variable $(\bullet)$. |
| $\varphi_i$ | Dual variable corresponding to current injection by line charging at $i^{th}$ bus. |
| $a_i$, $b_i$ and $c_i$ | Quadratic, linear and constant cost coefficients of generator at $i^{th}$ bus. |
| $a_{ij}$ | Transformer tap ratio as the proportion of turns of $i^{th}$ to $j^{th}$ bus. |
| $b_{lc_i}$ | Net line charging admittance connected at $i^{th}$ bus. |
| $C_{IV}$ | Cost function in IV-OPF model |
| $C_{PIV}$ | Cost function in P-IV-OPF model. |
| $e_i, f_i$ | Real and imaginary component of voltage at $i^{th}$ bus. |
| $G_i$ | Power injected by the generator at $i^{th}$ bus. |
| $g_{ij}$,$b_{ij}$ | Conductance and susceptance of line connected between $i^{th}$ and $j^{th}$ bus. |
| $g_{sh_i}$,$b_{sh_i}$ | Real and imaginary component of shunt at $i^{th}$ bus. |
| $I_{g_i}$ | Current injected by generator at $i^{th}$ bus. |
| $I_{ij}$ | Current flowing in line connected between $i^{th}$ and $j^{th}$ bus. |

| | |
|---|---|
| $I_{l_i}$ | Current injected by load at $i^{th}$ bus. |
| $I_{lc_i}$ | Line charging current at $i^{th}$ bus. |
| $I_{sh_i}$ | Shunt current at $i^{th}$ bus. |
| $L_i$ | Power drawn by the load at $i^{th}$ bus. |
| $RBL$ | Set of limiting line flows from receiving end buses. |
| $S_{ij}$ | Power flow in line connected between $i^{th}$ and $j^{th}$ bus. |
| $S_{lc_i}$ | Power injected by line charging at $i^{th}$ bus. |
| $S_{sh_i}$ | Power injected by shunt at $i^{th}$ bus. |
| $SBL$ | Set of limiting line flows from sending end buses. |
| $VLL$ | Set of buses with binding lower limit voltages. |
| $VUL$ | Set of buses with binding upper limit voltages. |

**Symbols used in Chapter 6**

| | |
|---|---|
| $\Lambda_i$ | LMP at bus $i$. |
| $\Lambda_i^C$ | Congestion component of LMP at bus $i$. |
| $\Lambda_i^E$ | Energy component of LMP at bus $i$. |
| $\Lambda_i^L$ | Loss component of LMP at bus $i$. |
| $\omega$ | A scalar quantity that represent an offset for transmission losses. |
| $D$ | The vector of demand. |
| $DF$ | A vector of loss distribution factor representing the distribution of losses in proportion to load at every bus. |
| $f_i$ | Cost function for each generator $(G_i)$ at $i^{th}$ bus. |

| | |
|---|---|
| $i$ and $l$ | bus and line index. |
| $L$ | The vector of loss. |
| $LF$ | The vector of loss factor. |
| $SF$ | Shift factor or power transfer distribution factor. |

**Symbols used in Chapter 7**

| | |
|---|---|
| $\gamma_i^R$ and $\gamma_i^I$ | Dual variable corresponding to real and reactive power balance equation at $i^{th}$ bus. |
| $\gamma_{\Im(loss)}^k$ | Reactive power loss component of LMP for $k^{th}$ load. |
| $\gamma_{\Re(loss)}^k$ | Real power loss component of LMP for $k^{th}$ load. |
| $\gamma_{cong}^k$ | Congestion component of LMP for $k^{th}$ load. |
| $\gamma_{lc}^k$ | Line charging component of LMP for $k^{th}$ load. |
| $\gamma_{loss}^k$ | Loss component of LMP for $k^{th}$ load. |
| $\gamma_{Pg}^k$ | Real power component of LMP for $k^{th}$ load. |
| $\gamma_{Qg}^k$ | Reactive power generation component of LMP for $k^{th}$ load. |
| $\gamma_{Ql}^k$ | Reactive power load component of LMP for $k^{th}$ load. |
| $\gamma_{sh}^k$ | Shunt component of LMP for $k^{th}$ load. |
| $\omega_{ij}^R$ and $\omega_{ij}^I$ | Marginal price corresponding to real and reactive loss component in line connected between $i^{th}$ and $j^{th}$ bus. |
| $\pi_{ij}^R$ and $\pi_{ij}^I$ | Marginal price corresponding to real and reactive congestion component in line connected between $i^{th}$ and $j^{th}$ bus. |
| $\Re(\bullet),\Im(\bullet)$ | Real and imaginary component of the variable $(\bullet)$. |

$e_i, f_i$ Real and imaginary component of voltage at $i^{th}$ bus.

$G_i$ Power injected by the generator at $i^{th}$ bus.

$i$ and $j$ Index represent the $i^{th}$ and $j^{th}$ bus.

$I_{g_i}$ Current injected by generator at $i^{th}$ bus.

$I_{ij}$ Current flowing in line connected between $i^{th}$ and $j^{th}$ bus.

$I_{l_i}$ Current injected by load at $i^{th}$ bus.

$I_{lc_i}$ Line charging current at $i^{th}$ bus.

$I_{sh_i}$ Shunt current at $i^{th}$ bus.

$ij$ Index represent the line connected between $i^{th}$ and $j^{th}$ bus.

$k$ Index represent the load at $k^{th}$ bus.

$L_i$ Power drawn by the load at $i^{th}$ bus.

$N$ The number of buses in the power system.

$S_{ij}$ Power flow in line connected between $i^{th}$ and $j^{th}$ bus.

**Acronyms / Abbreviations**

AC-OPF Alternating Current Optimal Power Flow

ARR Auction Revenue Right

CS Congestion Surplus

DC-OPF Direct Current Optimal Power Flow

DE Differential Evolution

EBE Equivalent Bilateral Exchange

| | |
|---|---|
| ES | Energy Surplus |
| FGR | Flowgate Right |
| FTR | Financial Transmission Right |
| HM | Hybrid Method |
| IV-OPF | Current and Voltage based Optimal Power Flow |
| KKT | Karush-Kuhn-Tucker |
| LBG | Linear Bottleneck Games |
| LC-DC-OPF | Loss Compensated Direct Current Optimal Power Flow |
| LMP | Locational Marginal Prices |
| LS | Loss Surplus |
| MPM | Marginal Participation Method |
| MS | Market-Settlement Surplus |
| MW | Mega-Watts |
| OPF | Optimal Power Flow |
| P-IV-OPF | Pseudo-Current and Voltage based Optimal Power Flow |
| PDLP | Primal-Dual Linear Programming |
| PF | Power Flow |
| PM | Proposed Method |
| PTDF | Power Transfer Distribution Factor |
| QCQP | Quadratically Constrained Quadratic Programming |

| | |
|---|---|
| SA | Silhouette Analysis |
| SMD | Standard Market Design |
| WRGI | Western Regional Grid of India |