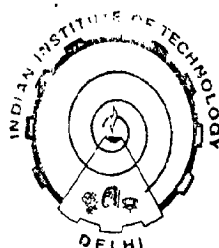


**ON MIXED FINITE ELEMENT METHODS FOR DIRICHLET  
PROBLEM OF FOURTH ORDER ELLIPTIC EQUATIONS WITH  
VARIABLE COEFFICIENTS**

by  
**S. BALASUNDARAM**

A Thesis submitted to the  
Indian Institute of Technology, Delhi  
for the award of the degree of  
**DOCTOR OF PHILOSOPHY**



DEPARTMENT OF MATHEMATICS  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
**MARCH, 1983**

Dedicated to  
THE MEMORIES OF  
MY MOTHER AND GRANDMOTHER

## CERTIFICATE

This is to certify that the thesis entitled 'On Mixed Finite Element Methods for Dirichlet Problem of Fourth Order Elliptic Equations with Variable Coefficients' which is being submitted by Mr. S. Balasundaram to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him under my guidance and supervision and has fulfilled all the requirements for the submission of this thesis.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.

( P.K. BHATTACHARYYA )  
Assistant Professor  
Department of Mathematics  
Indian Institute of Technology  
Hauz Khas, New Delhi-110016.

## ACKNOWLEDGEMENTS

It is my unique privilege to express my profound sense of gratitude to Dr. P.K. Bhattacharyya, for his valuable guidance, timely help and constant encouragement during the course of this research work.

Thanks are due to the Department of Mathematics, IIT Delhi, for the facilities provided for my work.

I thank the authorities of the Indian Institute of Technology, Delhi, for providing me with sufficient facilities for pursuing my research work and for the computer facilities provided.

I would be failing in my duties, if I do not recall with gratitude the encouragement and support generously given to me by my uncle, Mr. S.K. Somasundaram and aunt Mrs. Avadaiachi Somasundaram, which encouraged and enabled me to continue my higher education. I sincerely thank my brothers, sister and brother-in-law for their inspiration and encouragement throughout my studies.

Ambition of my father was to see me to acquire higher knowledge at every possible opportune time, I remember his advice and inspiring suggestions with reverence.

Mr. M. Ganapathi and Dr. K. Chidambaram made my life in hostel very pleasant and memorable. I owe a deep sense of gratitude not only to these friends but also to other numerous friends and well-wishers in Shivalik, who made my stay in the campus enjoyable. Mr. K. Pothiraj and Mr. Gopalasamy helped me in various ways during the course of the preparation of this thesis work. I thank them very much.

Miss Naalam Dhody, who did the excellent typing of the thesis deserves a special mention here.

S. BALASUNDARAM

## CONTENTS

|   | Page No. |
|---|----------|
| RESUME  | i        |
| CHAPTER 1: INTRODUCTION                                   | 1        |
| 1.1 The Relevance of the Problem of the Thesis            | 1        |
| 1.2 Sobolev Spaces  | 4        |
| 1.3 The State of the Art of Mixed Finite Element Analysis | 7        |
| CHAPTER 2: CONFORMING FINITE ELEMENT APPROXIMATIONS       | 17       |
| 2.1 Introduction  | 17       |
| 2.2 The Continuous Variational Problem                    | 17       |
| 2.3 Applications  | 32       |
| 2.4 Conforming Finite Element Analysis                    | 41       |
| 2.5 Discrete Problem                                      | 42       |
| 2.6 Error Estimate  | 43       |
| CHAPTER 3: MIXED FINITE ELEMENT METHOD-I                  | 47       |
| 3.1 Introduction  | 47       |
| 3.2 Mixed Method Formulation                              | 48       |
| 3.3 Examples  | 59       |
| 3.4 The Discrete Problems                                 | 62       |
| 3.5 Error Estimates of Finite Element Approximations      | 66       |

|   | Page No. |
|---|----------|
| CHAPTER 4: MIXED FINITE ELEMENT METHOD-II                                   | 82       |
| 4.1 Introduction  | 82       |
| 4.2 Mixed Method Formulation  | 84       |
| 4.3 Examples  | 95       |
| 4.4 Finite Element Approximation  | 97       |
| 4.5 Error Estimates   | 102      |
| <br>CHAPTER 5: MIXED FINITE ELEMENT METHOD-III                              | <br>117  |
| 5.1 Introduction  | 117      |
| 5.2 Mixed Method Formulation  | 119      |
| 5.3 Examples  | 129      |
| 5.4 Finite Element Approximation  | 137      |
| 5.5 Error Estimates   | 140      |
| <br>CHAPTER 6: MIXED FINITE ELEMENT METHOD-IV                               | <br>147  |
| 6.1 Introduction  | 147      |
| 6.2 Mixed Method Formulation  | 149      |
| 6.3 Examples  | 156      |
| 6.4 Finite Element Approximations   | 169      |
| 6.5 Error Estimates   | 161      |
| <br>CHAPTER 7: COMPUTER IMPLEMENTATIONS AND NUMERICAL<br>EXPERIMENTS        | <br>171  |
| 7.1 Introduction  | 171      |
| 7.2 Common Procedures   | 174      |
| 7.3 Computer Implementation Procedures<br>for Mixed Finite Element Method-I | 180      |
| 7.4 List of Subroutines   | 185      |

|   | Page No. |
|---|----------|
| 7.5 Numerical Experiments   | 186      |
| 7.6 Computer Implementation Procedures<br>for Mixed Finite Element Method-II  | 193      |
| 7.7 An Alternative Method   | 196      |
| 7.8 List of Subroutines for Method-II   | 203      |
| 7.9 Computer Implementation for Mixed<br>Finite Element Method-III            | 204      |
| 7.10 List of Subroutines for Method-III                                       | 206      |
| 7.11 Numerical Experiments  | 207      |
| 7.12 Computer Implementation Procedures<br>for Mixed Finite Element Method-IV | 212      |
| 7.13 An Alternative Method  | 216      |
| 7.14 List of Subroutines for Method-IV  | 218      |
| <br>CONCLUSIONS   | <br>219  |
| <br>BIBLIOGRAPHY  | <br>223  |
| <br>ADDITIONAL BIBLIOGRAPHY   | <br>233  |

---