

# **VISION-BASED CONTROL FOR ROBOTIC SYSTEMS: CONSTRAINT SATISFACTION AND RELATED PROBLEMS**

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# VISION-BASED CONTROL FOR ROBOTIC SYSTEMS: CONSTRAINT SATISFACTION AND RELATED PROBLEMS

by

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*Dedicated to friends and family...*

# Certificate

This is to certify that the thesis entitled “**Vision-Based Control for Robotic Systems: Constraint Satisfaction and Related Problems**”, submitted by **Chimmula Kishore** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Electrical Engineering, is a record of the original, bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of our knowledge.

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# *Abstract*

This thesis primarily explores the Image-Based Visual Servoing (IBVS) techniques and their applications in robotic control, emphasizing the need to address significant challenges. IBVS operates by continually assessing the positions of feature points within images, comparing them to their desired locations, and iteratively adjusting control laws until the error state converges to zero, signifying stability. However, this approach is not without its concerns. One such concern stems from the assumption that feature vectors remain perpetually visible in each camera frame, presuming that objects of interest remain within the camera's field of view (FoV). The practical scenario, where features move out of the FoV as robots progress towards their goals, is termed the visibility or FoV problem. Addressing this challenge is pivotal for the effectiveness of vision-based control systems. Furthermore, ensuring that control energy adheres to defined robot saturation limits while accommodating FoV constraints is imperative. This necessitates simultaneous consideration of FoV and input constraints in the design of IBVS control laws. Another important concern centers around the accuracy of depth estimates, which directly impact the image interaction matrix used in the IBVS control law. Inaccurate estimates can severely compromise control performance. Another critical issue is that IBVS predominantly employs kinematic controllers, which disregard dynamic effects from both robot and camera dynamics. These controllers assume instantaneous velocity attainment, but real-world scenarios demand dynamic modeling and control to handle uncertainties effectively. In situations where dynamic controllers are employed, numerical approximations are often used to estimate velocities from position measurements, introducing a delay into the system. A comprehensive understanding of delay-based analysis is crucial for understanding the behavior of such controllers. These challenges and considerations underpin the objectives of this thesis, aiming to maximize the potential of IBVS in robotic control applications. Inspired by human capabilities, this research advocates leveraging visual information from cameras to foster cooperative behaviors among robots. An application is presented in the form of leader-follower formation control. Through theoretical exploration and simulations, this research advances our comprehension of IBVS, offering practical solutions to its challenges while opening new avenues for exploiting visual information in real-world robotic control environments.



## सार

यह शोध प्रबंध मुख्य रूप से छवि-आधारित दृश्य सर्वोडिंग (IBVS) तकनीकों और उनके रोबोटिक नियंत्रण में अनुप्रयोगों का अन्वेषण करता है, महत्वपूर्ण चुनौतियों को संबोधित करने की आवश्यकता पर जोर देता है। IBVS छवियों के भीतर फीचर बिंदुओं की स्थिति का लगातार मूल्यांकन करके, उन्हें उनकी वांछित स्थितियों से तुलना करके, और नियंत्रण नियमों को तब तक समायोजित करके काम करता है जब तक कि त्रुटि स्थिति शून्य पर अभिसरित न हो जाए, जो स्थिरता का संकेत देता है। हालांकि, यह दृष्टिकोण अपने चिंताओं के बिना नहीं है। ऐसी एक चिंता इस धारणा से उत्पन्न होती है कि फीचर वेक्टर प्रत्येक कैमरा फ्रेम में लगातार दृश्यमान रहते हैं, यह मानते हुए कि रुचि की वस्तुएं कैमरे के दृश्य क्षेत्र (FoV) के भीतर रहती हैं। व्यावहारिक परिदृश्य, जहां फीचर अपने लक्ष्यों की ओर बढ़ने के साथ रोबोटों के दृश्य क्षेत्र से बाहर चले जाते हैं, को दृश्यता या FoV समस्या कहा जाता है। इस चुनौती का समाधान दृष्टि-आधारित नियंत्रण प्रणालियों की प्रभावशीलता के लिए महत्वपूर्ण है। इसके अलावा, FoV सीमाओं को समायोजित करते हुए नियंत्रण ऊर्जा को परिभाषित रोबोट संतृप्ति सीमाओं का पालन सुनिश्चित करना अनिवार्य है। यह IBVS नियंत्रण नियमों के डिजाइन में FoV और इनपुट बाधाओं के एक साथ विचार करने की आवश्यकता है। एक अन्य महत्वपूर्ण चिंता गहराई के अनुमान की सटीकता के इर्द-गिर्द घूमती है, जो सीधे IBVS नियंत्रण नियम में उपयोग की जाने वाली छवि इंटरैक्शन मैट्रिक्स को प्रभावित करती है। गलत अनुमान नियंत्रण प्रदर्शन को गंभीर रूप से प्रभावित कर सकते हैं। एक अन्य महत्वपूर्ण मुद्दा यह है कि IBVS मुख्य रूप से काइनेटिक नियंत्रकों को नियोजित करता है, जो रोबोट और कैमरा गतिशीलता दोनों से गतिशील प्रभावों को नजरअंदाज करते हैं। ये नियंत्रक तात्कालिक वेग प्राप्ति मानते हैं, लेकिन वास्तविक दुनिया के परिदृश्य अनिश्चितताओं को प्रभावी ढंग से संभालने के लिए गतिशील मॉडलिंग और नियंत्रण की मांग करते हैं। जिन स्थितियों में गतिशील नियंत्रकों का उपयोग किया जाता है, उन स्थितियों में स्थिति माप से वेग का अनुमान लगाने के लिए अक्सर संख्यात्मक अनुमानों का उपयोग किया जाता है, जो प्रणाली में एक देरी को परिचय देते हैं। ऐसे नियंत्रकों के व्यवहार को समझने के लिए देरी-आधारित विश्लेषण की व्यापक समझ महत्वपूर्ण है। ये चुनौतियाँ और विचार इस शोध प्रबंध के उद्देश्यों को रेखांकित करते हैं, जिसका उद्देश्य रोबोटिक नियंत्रण अनुप्रयोगों में IBVS की संभावनाओं को अधिकतम करना है। मानव क्षमताओं से प्रेरित होकर, यह शोध रोबोटों के बीच सहयोगात्मक व्यवहार को बढ़ावा देने के लिए कैमरों से दृश्य जानकारी का लाभ उठाने की वकालत करता है। नेता-अनुगामी गठन नियंत्रण के रूप में एक अनुप्रयोग प्रस्तुत किया गया है। सैद्धांतिक अन्वेषण और सिमुलेशन के माध्यम से, यह शोध IBVS की हमारी समझ को आगे बढ़ाता है, इसकी चुनौतियों के व्यावहारिक समाधान पेश करता है, जबकि वास्तविक दुनिया के रोबोटिक नियंत्रण वातावरण में दृश्य जानकारी के दोहन के नए रास्ते खोलता है।

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# Abbreviations

|             |                                                                |
|-------------|----------------------------------------------------------------|
| <b>VS</b>   | <b>V</b> isual <b>S</b> ervoing                                |
| <b>VSC</b>  | <b>V</b> isual <b>S</b> ervo <b>C</b> ontrol                   |
| <b>IBVS</b> | <b>I</b> mage <b>B</b> ased <b>V</b> isual <b>S</b> ervoing    |
| <b>PBVS</b> | <b>P</b> osition <b>B</b> ased <b>V</b> isual <b>S</b> ervoing |
| <b>FoV</b>  | <b>F</b> ield of <b>V</b> iew                                  |
| <b>PPC</b>  | <b>P</b> rescribed <b>P</b> erformance <b>C</b> ontrol         |
| <b>BLF</b>  | <b>B</b> arrier <b>L</b> yapunov <b>F</b> unction              |
| <b>DoF</b>  | <b>D</b> egree of <b>F</b> reedom                              |
| <b>WMR</b>  | <b>W</b> heeled <b>M</b> obile <b>R</b> obot                   |
| <b>CLF</b>  | <b>C</b> ontrol <b>L</b> yapunov <b>F</b> unction              |
| <b>QLF</b>  | <b>Q</b> uadratic <b>L</b> yapunov <b>F</b> unction            |
| <b>RRT</b>  | <b>R</b> apidly exploring <b>R</b> andom <b>T</b> ree          |
| <b>MPC</b>  | <b>M</b> odel <b>P</b> redictive <b>C</b> ontrol               |
| <b>UAV</b>  | <b>U</b> nmanned <b>A</b> erial <b>V</b> ehicle                |
| <b>UUB</b>  | <b>U</b> niformly <b>U</b> ltimately <b>B</b> ounded           |



# Symbols

|                      |                                                                               |
|----------------------|-------------------------------------------------------------------------------|
| $\mathbb{R}$         | set of all real numbers                                                       |
| $\mathbb{R}_+$       | set of positive real numbers including zero                                   |
| $\mathbb{R}_{>0}$    | set of positive real numbers                                                  |
| $\mathbb{R}^n$       | real space of dimension $n$                                                   |
| $\mathcal{L}_\infty$ | bounded function space                                                        |
| $\mathcal{F}_W$      | world reference frame                                                         |
| $\mathcal{F}_B$      | body/robot reference frame                                                    |
| $\mathcal{F}_C$      | camera reference frame                                                        |
| $\mathcal{F}_I$      | image reference frame                                                         |
| ${}^A\mathbf{R}_B$   | rotational transformation from frame $\mathcal{F}_B$ to frame $\mathcal{F}_A$ |
| ${}^A\mathbf{t}_B$   | position of origin of the frame $\mathcal{F}_B$ in frame $\mathcal{F}_A$      |
| $(X, Y, Z)$          | 3D world coordinates of a point                                               |
| $(x, y)$             | 2D normalized image coordinates of a point                                    |
| $(u, v)$             | 2D image pixel coordinates of a point                                         |
| $ a $                | absolute value of scalar $a$                                                  |
| $\ \mathbf{a}\ $     | Euclidean norm of vector $\mathbf{a}$                                         |
| $a \rightarrow b$    | $a$ goes to $b$                                                               |
| $a \in b$            | $a$ belongs to $b$                                                            |
| $a \triangleq b$     | $a$ is defined as $b$                                                         |