

# **SENSITIVITY ANALYSIS OF MULTICONDUCTOR TRANSMISSION SYSTEMS**

by  
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DEDICATED

TO

MY PARENTS

## ABSTRACT

In this thesis, the effects of various parameters on the characteristics of multiconductor transmission lines have been investigated using the technique of sensitivity analysis. First, the sensitivity theory for a single-phase line as well as a multi-phase transmission line is developed. A number of useful expressions, earlier not available in the literature on transmission lines, have been derived and presented. The theory is based on the sensitivity of eigenvalues and eigenvectors of the system propagation matrix. Making use of this theory, the sensitivities of receiving-end voltage and power of a typical EHV transposed and an untransposed line at power frequency are determined with respect to the various parameters of interest, such as spacing between the conductors, conductor radius, resistivity of conductor material, heights of conductors above earth plane, and, in case of bundle conductor lines, spacing between the sub-conductors.

The effects of load current, which lead to changes in sag and average height of the conductors, conductor temperature, and deflection of the subconductors, on carrier propagation in overhead transmission lines have been studied. The effects of the parameters of the terminal equipment, such as wave trap, coupling capacitor etc. on the carrier insertion loss have also been considered.

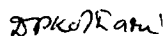
The theoretical work of Cristina and D'Amore<sup>4</sup> applied to overhead power transmission lines for predicting the performance of the

carrier channel has been extended successfully to underground cable systems. An accurate mathematical model which takes into account the conductor, sheath and skin effects is considered for this study. The matrix sensitivity functions of series-impedance and shunt-admittances matrices have been developed for various parameters of the cable system such as cable dimensions, spacing between the cables, cable depth, frequency, and conductor, sheath and earth resistivities, and the sensitivities of the modal attenuation constants and velocities at carrier frequencies have been evaluated. From the sensitivity values of the modal attenuation constants, the sensitivity of the actual attenuation with respect to several parameters is computed.

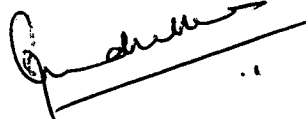
Since most EHV cable systems these days are crossbonded, the method of calculating the sensitivities in such systems is also described and results for a normally crossbonded scheme are given. Dang's model<sup>7</sup> has been used for studying the crossbonded cable. Since the series-impedance and shunt-admittance matrices using Dang's model, turn out to be the same for all the types of crossbonding schemes, it is sufficient to carry out the study for any one type of crossbonding scheme .

CERTIFICATE

Certified that the thesis entitled "Sensitivity Analysis of Multiconductor Transmission Systems", which is being submitted by Mr. Parmod Kumar, in partial fulfilment for the award of the Degree of Doctor of Philosophy in Electrical Engineering of the Indian Institute of Technology, Delhi, is a record of student's own work carried out by him under our joint supervision and guidance. The matter embodied in this thesis has not been submitted to any other University or Institute for the award of any other degree or diploma.



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