GROUP THEORETIC AND VARIATIONAL TECHNIQUES FOR THE SOLUTION OF NONLINEAR DIFFERENTIAL EQUATIONS OF PHYSICAL AND ENGINEERING SYSTEMS

G. CHANDRASEKARAN

A thesis submitted to the
Indian Institute of Technology, Delhi
for the award of the degree of
DOCTOR OF PHILOSOPHY

Department of Mathematics
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
MAY, 1988

DEDICATED TO....

MY FAMILY

AND

SUPERVISOR

CERTIFICATE

This to certify that the thesis entitled:

'GROUP THEORETIC AND VARIATIONAL TECHNIQUES FOR THE SOLUTION OF NONLINEAR DIFFERENTIAL EQUATIONS OF PHYSICAL AND ENGINEERING SYSTEMS',

which is being submitted by Mr. G.Chandrasekaran, Research Scholar, Mathematics Department to the Indian Institute of Technology, Delhi, for the award of the DEGREE OF DOCTOR OF PHILOSOPHY in MATHEMATICS, is a record of bonafide research work carried out by him under my guidance and supervision and has fulfilled all the requirements for the submission of this thesis.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.

O.P. BHUTANI
Professor,
Department of Mathematics
I.I.T., Delhi.

ACKNOWLEDGEMENT

I am profoundly grateful to Professor O.P. Bhutani, Department of Mathematics, Indian Institute of Technology, Delhi, for his valuable guidance throughout the preparation of this thesis. But for his generous encouragement, keen interest and kind cooperation, this work would not have been possible.

I take this opportunity to thank the authorities of Indian Institute of Technology, Delhi and Professor M.M. Chawla, Head of the Department of Mathematics, Indian Institute of Technology, Delhi, for providing all necessary facilities to undertake this work.

I also express my sincere thanks to Dr.(Mrs.) Poornima Mital for the fruitful discussions carried out during my research work.

It is my duty to thank Professor G.Shanmugam, Principal, Mepco Schlenk Engineering College, Sivakasi, Dr.B.Rajendran and Mr.P.Pandian, Department of Mathematics, M.S.E.C. for their kind cooperation and encouragement.

Thanks are also due to my colleagues Mr.K.Sankarakumar, Mr. K.Vijay Kumar, Mr. P. Sampath Narayanan, Mr. Vikas Bist, Mr. Sarbeswar Raut and others.

It is with pleasure, I mention my friends - Dr.S.Balasundaram, Dr. S. Gopalsamy, Dr. R. Sekar, Mr. S.Selvakumar, Mr.R.Subramanian, Mr. C.Thangaraj, Mr. A.Thayumanavan, Mr. R.Venkatraj and others who made my Shivalik Hostel life homely and enjoyable, during my stay in the IIT Campus.

Finally, Miss Neelam Dhody deserves praise for her intelligent and expert typing of this thesis and I sincerely thank her for the same.

G. CHANDRASEKARAN

SYNOPSIS

The thesis entitled "Group Theoretic and Variational Techniques for the Solution of Nonlinear Differential Equations of Physical and Engineering Systems" can be envisaged as consisting of two parts. First part, comprising of four chapters is concerned with the problems of Existence and Formulation of Variational Principles, first integrals and conservation laws via invariant variational principles and the Quasi-variational principles for nonlinear differential equations of physical and engineering systems. Second part, which also contains four chapters, deals with the invariant (similar) solutions of single/system of nonlinear partial differential equations and nonlinear initial value problems. We briefly outline the details of each of the chapters as follows:

CHAPTER-I: INTRODUCTION - THE SURVEY AND THE NECESSARY MATHEMATICAL TOOLS

Herein, the aim of this chapter is two fold in the sense that, besides giving a brief survey of the available literature related to our work in Chapters II-IX, we have given a brief resume of the consistency conditions for establishing the existence of variational principles for any single nonlinear ordinary differential equation of second order and for system of two second order nonlinear partial differential equations, indicating the procedure for writing down the functional

whenever it exists. Herein, we have also stated the classical Noether's theorem with special reference to Rund's invariance identities for finding the one-parameter infinitesimal transformation. Finally, we have given the short resume of Steinberg's symmetry method for finding similarity solutions of single/system of partial differential equations.

CHAPTER-II: EXISTENCE AND FORMULATION OF VARIATIONAL PRINCIPLES FOR SYSTEMS OF NONLINEAR DIFFERENTIAL EQUATIONS

In this chapter, we have utilized the consistency conditions given in Chapter-I, to study the existence and formulation of potential/alternate potential principles for the following systems of nonlinear differential equations of physical interest:

(i) Flows governed by the generalised Korteweg-de Vries (K-dV) equation,

(ii) Nonlinear lateral vibrations of a beam,

(iii) Nonlinear dispersive waves,

(iv) Time-dependent SU(2) Higg's model,

(v) The equation of a relativistic string,

(vi) Coupled system of nonlinear Klein-Gordon equations, and

(vii) Finite axisymmetric deformations of thin shells of revolutions.

CHAPTER-III: ON THE EXACT SOLUTIONS OF NONLINEAR DIFFERENTIAL EQUATIONS OF NUCLEAR ENGINEERING SYSTEM VIA INVARIANT VARIATIONAL PRINCIPLES

This chapter deals with the existence and formulation of the alternate potential principles for the nonlinear differential equations corresponding to the following physical situations of nuclear engineering system:

(i) The steady state heat conduction with nonlinear thermal conductivity and nonlinear source term, and

(ii) The reactor core optimization.

Further, the utilization of the invariance identities of Rund involving the Lagrangian and the generators of the infinitesimal Lie group and Noether's theorem has been made to obtain the first integral. Also, through the repeated application of the invariance under the transformation obtained, the exact solutions of the above mentioned differential equations have been generated for various choices of the parameters involved.

CHAPTER-IV: CONSERVATION LAWS FOR PHYSICAL SYSTEMS VIA INVARIANT VARIATIONAL PRINCIPLES

After proving the invariance under a one-parameter infinitesimal transformation group, we have used Noether's theorem for writing down the conservation laws for the following systems

of physical interest:

(i) Transonic gas flow equation,

(ii) (2+1)-dimensional nonlinear wave equation,

(iii) Flows governed by the generalised K-dV equation,

(iv) Generalized Duffing problem and

(v) Nonlinear wave equation of Klein-Gordon's type.

CHAPTER-V: QUASI-VARIATIONAL PRINCIPLE AND NONLINEAR WAVES

The purpose of this chapter is to develop quasi-variational principle to construct approximate solution to nonlinear wave problems governing the following physical situations:

(i) The variable coefficients K-dV equation and

(ii) The Benjamin-Bona and Mahoney (BBM) equation.

Here, the trial solution is chosen with the help of prior informations available. The solution found is slowly varying in the sense that higher order derivative terms are neglected.

CHAPTER-VI: INVARIANT SOLUTIONS OF SINGLE PARTIAL DIFFERENTIAL EQUATIONS BY SYMMETRY METHOD

Herein, we have utilized the Steinberg's symmetry method for generating the invariant (similar) solutions of

the following nonlinear partial differential equations of physical interest:

(i) The generalised Boussinesq equation,

(ii) The generalised Duffing problem, and

(iii) The nonlinear wave equation of Klein-Gordon's type.

Possible invariant solutions of these equations for physically realizable forms of the parameters involved have been obtained.

CHAPTER-VII: INVARIANT SOLUTIONS OF GENERALISED KORTEWEG-DE VRIES-BURGERS TYPE EQUATIONS

As in Chapter-VI, we have utilized the symmetry method to the generalised Korteweg-de-Vries-Burgers type equations to generate the invariant solutions. Possible invariant solutions for physically realizable forms of the parameters involved are obtained.

CHAPTER-VIII: INVARIANT SOLUTIONS OF SYSTEMS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Unlike Chapters VI-VII, wherein we had generated the invariant solutions of single nonlinear partial differential equations, we have in this chapter, utilized the symmetry method for obtaining the invariant solutions of the following systems of nonlinear partial differential equations of physical

interest:

(i) Longitudinal waves in a nonlinear geometrically dispersive solid and

(ii) Generalised coupled K-dV equation.

CHAPTER-IX: CONTINUOUS TRANSFORMATION GROUPS AND SERIES SOLUTION OF NONLINEAR INITIAL VALUE PROBLEMS

In this chapter, we have utilized the theory of continuous transformation groups, to construct the series form of the solution of the following initial value problems of physical interest, with the associated infinitesimal generator of the dynamical systems:

(i) Oscillator with nonlinear viscous resistance,

(ii) The Vander Pol equation, and

(iii) The Duffing equation.

Tedious algebra associated with the substitution of power series with unknown coefficients and solution of nonlinear algebraic equations in solving an IVP is thereby eliminated.

The following papers which form the part of the thesis have been published/submitted:

1) On The Exact Solutions of Nonlinear Differential Equations of Nuclear Engineering System Via Invariant Variational Principles. Int. J. Engng. Sci. 26(3) 243-248 (1988).

2) On Invariant Solutions of the Generalised Boussinesq Equation. Int. J.Engng. Sci. 26(3) 307-310 (1988).

3) Existence and Formulation of Variational Principles for Systems of Differential Equations. Presented at the 31st Congress of ISTAM, Gwalior (1986).

4) Conservation Laws for Physical Systems via Invariant Variational Principles. Presented at the 31st Congress of ISTAM , Gwalior (1986).

5) On Invariant Solutions of the Generalised Korteweg-de-Vries-Burgers Type Equations-II. Communicated to J.Math. Phys.

CONTENTS
