

**VIBRATION AND DAMPING ANALYSIS OF MULTILAYERED
PLATES AND SHELLS**

By
NAIYAR ALAM
Department of Mechanical Engineering

Submitted
in fulfilment of the requirements
of the degree of
Doctor of Philosophy
to the

Indian Institute of Technology, Delhi

August, 1980

TO MY PARENTS

CERTIFICATE

This is to certify that the thesis entitled 'Vibration and Damping Analysis of Multilayered Plates and Shells', being submitted by Mr. Naiyar Alan to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy in Mechanical Engineering is a record of bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for submission of this Thesis, which has reached the requisite standard.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.

**(N.T. Asnani)
Assistant Professor
Mechanical Engineering Department
Indian Institute of Technology
Hauz Khas, New Delhi-110016.**

ACKNOWLEDGEMENTS

The author is deeply indebted to Dr. N.T. Ashani, Assistant Professor, Department of Mechanical Engineering, who suggested the problem and supervised the work reported in this thesis. His unceasing enthusiasm, fruitful discussions and constant encouragement inspired the author at all stages of this work.

The author expresses his thanks to Dr. B.C. Nakra, Professor and Dr. H.B. Mathur, Professor and Head, Department of Mechanical Engineering, I.I.T. Delhi for providing various facilities for carrying out this work.

The author wishes to record his deep sense of gratitude to Dr. S.H. Mohsin, Professor, Department of Mechanical Engineering, A.M.U. Aligarh for his affectionate encouragements at all stages of this work.

The author is grateful to his friend Mr. Tariq Aziz for giving valuable suggestions and for providing moral support and help at various difficult stages during the course of this work.

Financial support for the present work was made available under centrally sponsored scheme (Q.I.P.) by Ministry of Education, Government of India and study leave was granted by Aligarh Muslim University. These are duly acknowledged.

The author presents his hearty thanks and sincere apologies to his near and dear ones who very willingly endured certain hardships which resulted from the author's preoccupation with the present work.

A special word of thanks for Mr. D.R. Joshi for his painstaking typing.

NAIYAR ALAM

ABSTRACT

The governing equations of motion for vibrations of a general multilayered rectangular plate having arbitrary number of alternate stiff and soft layers have been derived, using variational principles. Material of all the layers has been taken to be orthotropic. As in general conventional sandwich structures, extension and bending deformations in stiff layers and only shear deformations in soft layers have been considered. In addition to transverse inertia, longitudinal translatory and rotary inertias of the plate have been included. Vibration and damping analysis of a multilayered simply supported plate consisting of alternate elastic and viscoelastic layers has been carried out using corresponding principle of linear viscoelasticity. Damping effectiveness in terms of loss-factor for all families of modes of vibration for three, five, seven and nine layered plates has been evaluated under constant size criterion and its variation with geometric and material property parameter has been studied.

In case of multilayered rectangular plates consisting of all stiff layers, the analysis of plate with alternate stiff and soft layers discussed above is not valid. As such more refined analysis which considers bending, extension and shear deformations in all the layers of multilayered plate

has been carried out. This analysis also includes rotary, longitudinal translatory and transverse inertias of the plate. Material of each layer has been considered to be orthotropic. The results for vibration resonant frequencies and associated loss-factors for three, five and seven layered rectangular plates obtained by two analyses are compared. Damping analysis of multicored rectangular plate consisting of a pair of outer elastic faces and a number of cores has been carried out by using the refined analysis.

The governing equations of motion for vibration of a general multilayered cylindrical shell having arbitrary number of orthotropic material layers have been derived, using variational principles. In this analysis all deformations in the layers and all inertias of the shell have been considered as in the refined analysis of plates. Damping effectiveness in terms of system loss-factor for all families of modes of vibration for three, five and seven layered, constant thickness radially simply supported shell, with alternate stiff and soft viscoelastic layers has been evaluated and its variation with geometric and material properties parameters has been studied for axisymmetric and anti-axisymmetric vibrations.

The refined analysis of plate and shell has been extended for vibration and damping analysis of rectangular

plates and shells of fibre reinforced composite materials.

These studies have been carried out by changing geometrical and material properties of composite laminates with different stacking arrangements under constant thickness criterion, for various fibre orientations.

CONTENTS

	Page No.
CERTIFICATE	i
ACKNOWLEDGEMENTS	ii
ABSTRACT	iii
CONTENTS	vi
TERMINOLOGY	xi
 CHAPTER 1	
1.1 Introduction	1
1.2 Literature Survey	3
1.3 Outline of Present Work	15
 CHAPTER 2	
VIBRATION AND DAMPING ANALYSIS OF MULTI- LAYERED RECTANGULAR PLATES WITH CONSTRAINED VISCOELASTIC LAYERS	19
2.1 Introduction	19
2.2 Governing Equations of Motion	20
2.2.1 Assumptions	20
2.2.2 Derivation of Equations of Motion	21
2.3 Solution for Vibration of Multilayered Plate with Simply Supported Edges	36
2.3.1 Solution of Multilayered Plate with All Elastic Layers	37
2.3.2 Resonance Frequencies and Associated Loss-Factors for a Multilayered Plate with Constrained Viscoelastic Layers	49
2.4 Computer Program	52
2.5 Results and Discussion	52
2.5.1 Displacement Ratios for Various Families of Modes	53
2.5.2 Verification with Some Reported Results	56
2.5.3 Variation of Resonance Frequency Parameter λ and Associated System Loss-Factor η_s with Geometric and Material Property Parameters of Multilayered Symmetric Plate	62

2.5.3.1	Variation of λ and η_0 with δ	63
2.5.3.2	Variation of λ and η_0 with T/a	63
2.5.3.3	Variation of λ and η_0 with V	66
2.5.3.4	Variation of λ and η_0 with γ	68
2.5.4	Effect of Unsymmetry on η_0 of Multilayered Plate	69
2.5.5	Maximization of System Loss-Factor for Multilayered Plate	71
2.5.6	Variation of η_0 with λ	76
2.5.7	Effect of Variation of Material Loss-Factor of Viscoelastic Layers on η_0	80
CHAPTER 3	VIBRATION AND DAMPING ANALYSIS OF MULTILAYERED RECTANGULAR PLATE (REFINED ANALYSIS)	122
3.1	Introduction	122
3.2	Derivation of Equations of Motion	123
3.3	Solution for Vibration of Multilayered Plate with Simply Supported End Conditions	133
3.3.1	Solution of Multilayered Plate with All Elastic Layers	134
3.3.2	Resonance Frequencies and Associated Loss-Factors for a Multilayered Plate with Viscoelastic Layers	142
3.4	Computer Program	143
3.5	Results and Discussion	146
3.5.1	Displacement Ratios for Various Families of Modes	147
3.5.2	Resonance Frequencies and Associated System Loss-Factors for Five and Seven Layered Plates	150
3.5.3	Comparison of Results with Refined and Conventional Analysis	157
3.5.3.1	Effect of Variation of δ of Viscoelastic Layers	158
3.5.3.2	Effect of Variation of Thickness Ratio V	160

	3.5.3.3 Effect of Unsymmetry	161
	3.5.4 Damping of Multicored Plate	162
CHAPTER 4	VIBRATION AND DAMPING ANALYSIS OF MULTILAYERED CYLINDRICAL SHELL	181
4.1	Introduction	181
4.2	Derivation of Equations of Motion	182
4.3	Solution for Finite Multilayered Shell with Simply Supported Ends	193
4.3.1	Solution of Multilayered Shell with All Elastic Layers	193
4.3.2	Resonance Frequencies and Associated System Loss-Factors for a Multilayered Shell with Alternate Elastic and Viscoelastic Layers	200
4.4	Axiallysymmetric Vibration of Multilayered Cylindrical Shell	202
4.4.1	Solution for Simply Supported Ends with All Elastic Layers	204
4.4.2	Resonance Frequencies and Associated System Loss-Factors for a Multilayered Shell with Alternate Elastic and Viscoelastic Layers	208
4.5	Computer Program	210
4.6	Results and Discussion for Axiallysymmetric Vibration of Multilayered Shell	210
4.6.1	Displacement Ratios for Various Families of Modes	211
4.6.2	Vibration and Damping Analysis of Axiallysymmetric Vibration of Two Layered Cylindrical Shell	213
4.6.3	Variation of λ and η_b with Geometric and Material Property Parameters of a Constant Size Multilayered Shell	215
4.6.3.1	Variation of λ and η_b with β_1	216
4.6.3.2	Variation of λ and η_b with V	218
4.6.3.3	Variation of λ and η_b with δ	219
4.6.3.4	Variation of λ and η_b with T/a_1	220

4.7	Results and Discussion for Anti-symmetric Vibration of Multilayered Cylindrical Shell	221
4.7.1	Displacement Ratios for Various Families of Modes	222
4.7.2	λ and η_j with Different Values of 'j'	225
4.7.3	Variation of Frequency and System Loss-Factor of a Three Layered Sandwich Shell	225
4.7.4	Variation of λ and η_j with Geometric and Material Property Parameters of a Constant Size Multilayered Shell	228
4.7.4.1	Variation of λ and η_j with β_1	229
4.7.4.2	Variation of λ and η_j with V	230
4.7.4.3	Variation of λ and η_j with δ	231
4.7.4.4	Variation of λ and η_j with T/h_1	232
CHAPTER 5	VIBRATION AND DAMPING ANALYSIS OF COMPOSITE MATERIAL PLATES AND SHELLS	262
5.1	Introduction	262
5.2	Laminates of Composite Materials	264
5.3	Results and Discussion	269
5.3.1	Comparison with Reported Results for Plate	269
5.3.2	Optimal Design of Laminated Composite Plate for Maximization of Flexural Frequency	272
5.3.3	Vibration and Damping Analysis of Cross-Ply Antisymmetric Laminated Composite Plate	290
5.3.4	Vibration and Damping Analysis of Anisymmetric Vibration of Composite Cylindrical Shell	291
5.3.5	Vibration and Damping Analysis of Anisymmetric Vibration of Cross-Ply Antisymmetric Laminated Cylindrical Shell	294

CHAPTER 6	CONCLUDING REMARKS	308
	REFERENCES	318
	APPENDIX I	333
	APPENDIX II(a)	343
	APPENDIX II(b)	350
	APPENDIX III	357
	APPENDIX IV(a)	366
	APPENDIX IV(b)	385
	APPENDIX IV(c)	394