

SYNCHRONIZATION OF COUPLED BENCHMARK OSCILLATORS: THEORY AND EXPERIMENTS

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SYNCHRONIZATION OF COUPLED BENCHMARK OSCILLATORS: THEORY AND EXPERIMENTS

by

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Dedicated to

Smt. Usha Joshi and Dr. Gopal Krishan Joshi

Certificate

This is to certify that the thesis entitled “**SYNCHRONIZATION OF COUPLED BENCHMARK OSCILLATORS: THEORY AND EXPERIMENTS**”, submitted by **Shyam Krishan Joshi** to the Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy in Electrical Engineering, is a record of the original, bonafide research work carried out by him under our supervision and guidance. The thesis has reached, to the best of our understanding, the standards fulfilling the requirements of the regulations related to the award of the degree. The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma to the best of our knowledge.

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Abstract

Synchronization is a fundamental phenomenon in the collective dynamics of coupled oscillators whereby oscillators with different phases, when coupled, adjust their rhythm to a common phase. Examples include synchronization of pacemaker cells in the heart, synchronization of physiological rhythms, cohesive flashing of fireflies, clock synchronization in decentralized computing, frequency synchronization in complex power networks, crowd synchrony on London's Millennium Bridge, and synchronization of neural oscillators for perceptual decision making. The Kuramoto model is the most popular model to investigate the synchronization phenomena in coupled oscillators. Various researchers have found sufficient coupling gains that yield synchronization of Kuramoto oscillators. While synchronization in the one dimensional Kuramoto oscillators has been studied, similar issues for synchronization of higher-order oscillators are relatively unclear. This thesis investigates sufficient coupling gains for the synchronization of higher-order oscillators. We address this issue for benchmark oscillators using a combination of theoretical methods, such as the Lyapunov stability theorem, LaSalle's invariance principle, concepts of non-smooth Lyapunov functions, upper Dini-derivatives, and basic graph-theoretic notions of Laplacian matrix and Spanning trees, as well as numerical simulations and experimental testing. The primary contributions of this thesis are,

- Synchronization analysis has been performed using quadratic Lyapunov function and non-smooth Lyapunov function approaches for the second-order Van der Pol and Fitzhugh Nagumo oscillators. The cases of all-to-all topology and arbitrary connected network topologies with

linear diffusive couplings have been investigated to determine the sufficient coupling gains for synchronization. It is observed that synchronization would occur if the coupling gain between oscillators is sufficiently large. The results have been confirmed through simulations.

- Electronic set-ups for the Van der Pol oscillators and the Fitzhugh Nagumo oscillators have been fabricated and tested for synchronization. The cases of three all-to-all networked oscillators and four-star networked oscillators have been covered for each of the above types of oscillators to investigate the practical threshold of coupling gain synchronization. Possible reasons for discrepancies in analytical and experimental results are discussed.

- Networks of third-order oscillators, namely the Hindmarsh Rose oscillator, the Goodwin oscillator, and the Ring oscillator, have been investigated to achieve synchronization using the non-smooth Lyapunov function approach. The cases of all-to-all and arbitrary connected oscillators' networks have been covered for each of these third-order oscillators. It was found that these oscillators synchronize for coupling gains more than some threshold values. Simulations have confirmed these results. Next, experimental electronic set-up for three all-to-all coupled Hindmarsh Rose oscillators and four all-to-all networked Ring oscillators were developed. Again, it was observed that synchronization occurs in these third-order benchmark oscillators with single state linear diffusive couplings when coupling gain is more than some threshold.

- The pinning control of coupled second-order oscillators has been investigated for independent networks of Van der Pol as well as Fitzhugh Nagumo oscillators with all-to-all as well as arbitrary connected network topologies using quadratic Lyapunov function approach. Simulations confirm the results. Electronic experimental set-ups have been fabricated for these oscillators, and synchronization is practically achieved by using a single pinning controller. Overall, this thesis presents case studies of synchronization of coupled second-order and third-order benchmark oscillators that may be relevant for understanding the consensus phenomena in complex oscillatory networks.

सार

सिंक्रोनाइजेशन युग्मित दोलक की सामूहिक गतिकी में एक मूलभूत घटना है जिससे विभिन्न चरणों वाले दोलक, जब युग्मित होते हैं, तो अपनी लय को एक सामान्य चरण से समायोजित करते हैं। उदाहरणों में दिल में पेसमेकर कोशिकाओं का सिंक्रनाइजेशन, सिंक्रनाइजेशन शामिल हैं शारीरिक लय, फायरफ्लाइज़ की कोइशिव चमक, विकेन्द्रीकृत कंप्यूटिंग में घड़ी का सिंक्रनाइजेशन, जटिल विद्युत नेटवर्क में आवृत्ति तुल्यकालन, लंदन पर भीड़ समकालिकता मिलेनियम ब्रिज, और अवधारणात्मक निर्णय लेने के लिए तंत्रिका दोलक का सिंक्रनाइजेशन। युग्मित दोलक में कूर्मोटो मॉडल सिंक्रनाइजेशन घटना की जांच करने के लिए सबसे लोकप्रिय मॉडल है। विभिन्न शोधकर्ताओं ने पर्याप्त युग्मन लाभ पाया है जो कूर्मोटो ऑसिलेटर्स की सिंक्रनाइजेशन का उत्पादन करते हैं। जबकि एक आयामी कूर्मोटो में सिंक्रनाइजेशन थरथरानवाला का अध्ययन किया गया है, उच्च-क्रम के थरथरानवाला के सिंक्रनाइजेशन के लिए समान मुद्दे अपेक्षाकृत अस्पष्ट हैं। यह थीसिस उच्च-क्रम दोलक सिंक्रनाइजेशन के लिए पर्याप्त युग्मन लाभ की जांच करता है। हम एक संयोजन का उपयोग करते हुए बेंचमार्क ऑसिलेटर के लिए इस मुद्दे को संबोधित करते हैं। सैद्धांतिक विधियाँ, जैसे कि लैपुनोव स्थिरता प्रमेय, लासेल के आक्रामक सिद्धांत, गैर-चिकनी लैपुनोव कार्यों की अवधारणाएं, ऊपरी दीनी-डेरिवेटिव और बुनियादी ग्राफ-सिद्धांत लाप्लासियन मैट्रिक्स और स्पैनिंग पेडों की धारणाएँ, साथ ही संख्यात्मक सिमुलेशन और प्रयोगात्मक परीक्षण। इस थीसिस के प्राथमिक योगदान हैं,

- द्विघात का उपयोग करते हुए सिंक्रोनाइजेशन विश्लेषण किया गया है दूसरे क्रम के वैन डेर पोल और फिटज़्जुघ नागुमो के लिए ल्यपुनोव समारोह दृष्टिकोण दोलन। सभी के लिए सभी टोपोलॉजी और रैखिक डिफ्यूसिव कपलिंग के साथ मनमाने ढंग से जुड़े नेटवर्क टोपोलॉजी के मामलों की जांच की गई है ताकि पर्याप्त युग्मन लाभ का निर्धारण किया जा सके तादात्म्य। यह देखा गया है कि अगर युग्मन के बीच लाभ होता है तो तुल्यकालन होता है थरथरानवाला पर्याप्त रूप से बड़ा है। परिणामों की पुष्टि सिमुलेशन के माध्यम से की गई है।

- वैन डेर पोल ऑसिलेटर्स और फिटज़्जुघ नागुमो ऑसिलेटर्स के लिए इलेक्ट्रॉनिक सेट-अप तुल्यकालन के लिए गढ़े और परीक्षण किए गए हैं। सभी नेटवर्क के लिए तीन के मामले उपरोक्त प्रकारों में से प्रत्येक के लिए ऑसिलेटर और फोर-स्टार नेटवर्क ऑसिलेटर कवर किए गए हैं थरथरानवाला युग्मन लाभ तुल्यकालन के व्यावहारिक सीमा की जांच करने के लिए। मुमकिन विश्लेषणात्मक और प्रयोगात्मक परिणामों में विसंगतियों के कारणों पर चर्चा की जाती है।

- तीसरे क्रम के थरथरानवाला के नेटवर्क, जिसका नाम हिंदमर्श रोज ऑसिलेटर, गुडविन है थरथरानवाला, और रिंग थरथरानवाला, का उपयोग कर सिंक्रनाइजेशन प्राप्त करने के लिए जांच की गई है गैर-चिकनी लैपुनोव फ़ंक्शन दृष्टिकोण। ऑल-टू-ऑल और अनियंत्रित कनेक्टेड ऑसिलेटर के मामले इनमें से प्रत्येक तीसरे क्रम के ऑसिलेटर के लिए नेटवर्क को कवर किया गया है। ऐसा पाया गया कि ये थरथरानवाला कुछ सीमा मूल्यों से अधिक युग्मन लाभ के लिए सिंक्रनाइज़ करते हैं। सिमुलेशन इन परिणामों की पुष्टि की है। इसके बाद, तीन सभी

के लिए प्रयोगात्मक इलेक्ट्रॉनिक सेट-अप सभी युग्मित हिण्डमार्स रोज़ और सभी चार-सभी नेटवर्क रिंग ऑसिलेटर विकसित किए गए थे। फिर, यह देखा गया कि इन तीसरे क्रम के बेंचमार्क ऑसिलेटर में सिंक्रोनाइज़ेशन होता है। जब एकल लाभ कुछ सीमा से अधिक हो तो एकल राज्य रैखिक विसरित युग्मन के साथ।

· स्वतंत्र के लिए युग्मित दूसरे क्रम के ऑसिलेटर्स के पिनिंग नियंत्रण की जांच की गई है वैन डेर पोल के नेटवर्क के साथ-साथ फिटजूघ नागुमो ऑसिलेटर के साथ-साथ सभी के लिए द्विघात ल्यपुनोव फ़ंक्शन दृष्टिकोण का उपयोग करके मनमाने ढंग से जुड़े नेटवर्क टोपोलॉजी के रूप में। सिमुलेशन परिणामों की पुष्टि करें। इनके लिए इलेक्ट्रॉनिक प्रायोगिक सेट-अप तैयार किए गए हैं थरथरानवाला, और तुल्यकालन व्यावहारिक रूप से एक एकल पिनिंग कंट्रोलर का उपयोग करके प्राप्त किया जाता है।

कुल मिलाकर, यह थीसिस युग्मित दूसरे क्रम और तीसरे क्रम के सिंक्रोनाइज़ेशन के मामले के अध्ययन को प्रस्तुत करता है बेंचमार्क ऑसिलेटर जो सर्वसम्मति की घटनाओं को समझने के लिए प्रासंगिक हो सकता है जटिल ऑसिलेटरी नेटवर्क में।

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Abbreviations

Abbreviation/Symbol	Description
QLF	Quadratic Lyapunov function
NSLF	Non-smooth Lyapunov function
PCB	Printed circuit board
D	Degree matrix
A	Adjacency matrix
L	Laplacian matrix
K_p	Uniform coupling gain
K_{pi}	Non-uniform coupling gain
D^+	Upper Dini-derivative