

DYNAMICS OF INDUCED MAPS ON HYPERSPACES

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CERTIFICATE

I am satisfied that the thesis entitled *Dynamics of Induced Maps on Hyperspaces* presented by Mr. Puneet Sharma (2005MAZ8111) is worthy of consideration for the award of the degree of Doctor of philosophy and is a record of the original bonafide research work carried out by him under my guidance and supervision and the results contained in it have not been submitted in part or full to any other university or Institute for award of any degree/diploma.

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ABSTRACT

The study of set-valued dynamics originated in 1975 when Bauer and Sigmund studied the relation between the topological entropy of (X, T) and $(\mathcal{K}(X), T_{\mathcal{K}})$, where X is a compact metric space T is a homeomorphism on X , $\mathcal{K}(X)$ is the set of all non-empty compact subsets of X and $T_{\mathcal{K}}$ is the homeomorphism on $\mathcal{K}(X)$ generated by T in a suitable way. They also studied the relation between topological transitivity (resp. weak mixing, topological mixing and expansivity) of the two systems.

In this thesis, we study the dynamical behaviour of the induced system on the hyperspaces, for a given discrete topological dynamical system. In the process, we obtain conditions on the endowed hyperspace topology under which the dynamical behaviour of the map on the base space is inherited by the induced map on the hyperspace. We study the relation between properties like dense set of periodic points, transitivity, weakly mixing, topological mixing, existence of horseshoe for the map f on the base space and $\bar{f}$ on the hyperspace. Several of the known results come up as corollaries to our results. We also discuss some metric related dynamical properties on the hyperspace. We prove that all forms of sensitivities of $(\mathcal{K}(X), \bar{f})$ partly imply the same for (X, f) , although the converse holds in some cases. We also study the relation between some of the metric dependent properties like equicontinuity, expansivity, existence of scrambled sets etc. for the map f on the base space and $\bar{f}$ on the hyperspace.

Further, we study a special class of systems known as “Shift Spaces”. We compare the dynamics of the shift system (Ω, σ) with its induced counterpart $(\mathcal{K}(\Omega), \bar{\sigma})$. We study the full shift, subshift of finite type or any arbitrary subshift and compare their dynamics with that of their induced maps on the hyperspace. We prove that, for transitive subshifts, cofinite sensitivity is equivalent to syndectic sensitivity. In the process we prove that all sturmian subshifts are cofinitely sensitive.

Finally, we study the dynamics displayed by the combined effect of the join of distinct dynamical systems. In the process, we define some new notions of dynamics displayed by the system. We compare the various dynamical properties of such a relation with its individual defining components as well as its induced counterpart on the hyperspace. Some comparisons can be easily derived, but this study also leads to various questions that have been proposed.

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