

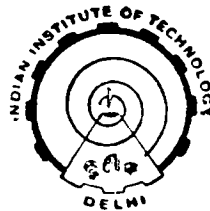
**SOME STUDIES ON THE ELASTODYNAMIC  
BEHAVIOUR OF UNBALANCED AND  
COUNTERWEIGHTED FOUR-BAR MECHANISMS**

by

**A BALASUBRAMONIAN**

Submitted  
in fulfilment of the requirements  
of the degree of

**DOCTOR OF PHILOSOPHY**



**Department of Mechanical Engineering**  
**INDIAN INSTITUTE OF TECHNOLOGY, DELHI**  
September 1986

TO  
MY PARENTS  
AND  
MY TEACHERS

CERTIFICATE

Certified that the thesis entitled "Some Studies on the Elastodynamic Behaviour of Unbalanced and Counterweighted Four-Bar Mechanisms", which is being submitted by Shri A. Balasubramanian for the award of the degree of Doctor of Philosophy in Mechanical Engineering, is a record of bona fide research work carried out by him under my supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.



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