MICROSTRUCTURE BASED ELASTOPLASTIC BEHAVIOUR OF POROUS MATERIALS WITH SNOW AS AN EXAMPLE

by

Anurag Kumar Singh

Department of Applied Mechanics

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Dedicated to my beloved parents and family

CERTIFICATE

This is to certify that the thesis entitled **“MICROSTRUCTURE BASED ELASTOPLASTIC BEHAVIOUR OF POROUS MATERIALS WITH SNOW AS AN EXAMPLE”** being submitted by **Mr. Anurag Kumar Singh** is the report of bonafide research work carried by him under my supervision. This thesis has been prepared in conformity with the rules and regulations of the INDIAN INSTITUTE OF TECHNOLOGY DELHI. I further certify that the thesis has attained a standard required for the award of **DOCTOR OF PHILOSOPHY** degree of the institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.

Navin Kumar

Prof. Puneet Mahajan
Department of Applied Mechanics
Indian Institute of Technology Delhi
New Delhi-110016, Delhi, India

Prof. Navin Kumar
Department of Mechanical Engineering
Indian Institute of Technology Ropar
Rupnagar- 140001, Punjab, India

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ABSTRACT

Snow is a porous material with network of interconnected sintered ice crystals. The mechanical behaviour of snow depends upon its density, arrangement of ice network or microstructure, and deformation behaviour of ice. The microstructure of snow may be anisotropic and this can lead to an anisotropy in its stress-strain relation. The anisotropy in microstructure of snow can be expressed mathematically in terms of a second rank fabric tensor. Here, we present a stress-strain relation for snow which has dependence on microstructure built in it through the fabric tensor.

Relations for fabric-based elastic and strength properties of snow have successfully estimated recently. Motivated by this, we propose a fabric-based macroscopic elasto-plastic constitutive law for snow, which can be used to study avalanche initiation. The fabric tensor and density-dependent yield surface with a provision for isotropic hardening/softening are used in this process. Beyond the initial yield, the yield function grows till the strength of the snow is reached and then softens. Since snow exhibits tension and compression behavior asymmetry, a piece-wise quadratic yield function is used.

The study is performed over different density snow samples comprising of round grain (RG), faceted crystal (FC), and depth hoar (DH) snow classes. Mean Intercept Length based fabric tensor (MIL fabric) for each snow sample is determined from the X-ray micro-computed tomographic (μCT) data. The μCT data is employed to construct the 3D μFE model of each snow sample. Each sample's homogenised stress-strain response and mechanical properties are determined from their 3D μFE model subjected to different boundary conditions. The unknown parameters of the fabric-elasticity and fabric-strength model are evaluated from the snow samples' homogenised elastic, strength, and fabric data. The constant of isotropic hardening/softening function is determined from the homogenised stress-strain and accumulated plastic strain data of snow samples.

The fabric-based failure surface in 3D, biaxial normal-normal and normal-shear are constructed, which depicts the strength asymmetry in tension and compression. The macroscopic constitutive law has been implemented as FE code (VUMAT) to predict the fabric-based stress-strain response. The macroscopic constitutive law and µFE based stress-strain response for high-density RG, FC, DH, and low-density FC snow in uniaxial and confined compression are compared. The fabric and µ-FE data showed a good match.

The macroscopic constitutive law is applied to layered snowpack with strong and weak layers. Recent studies on snowpack, such as subjected to skier and gravity load, Propagation Saw Test (PST), are investigated with the fabric-based model. The stress distribution in a weak layer at the different slab and weak layer thicknesses, and PST output viz. critical crack length, slab failure, fracture propagation speed, and fracture propagation distance are predicted from the macroscopic law-based investigations and compared with the reported data. The reported and macroscopic law-based data matches reasonably. The effect of fabric on the mechanical behaviour in the layered snowpack is shown and compared with the initial fabric value-based analysis.

सारांश

हिम एक झरझरा पदार्थ है जिसमें आपस में जुड़े हुए बर्फ क्रिस्टल के नेटवर्क होते हैं। हिम का यांत्रिक व्यवहार उसके घनत्व, बर्फ नेटवर्क या सूक्ष्म संरचना की व्यवस्था और बर्फ के विरूपण व्यवहार पर निर्भर करता है । हिम की सूक्ष्म संरचना अनिसोट्रोपिक हो सकती है, और इससे इसके स्ट्रेस-स्ट्रेन संबंध में अनिसोट्रॉपी हो सकती है। हिम की सूक्ष्म संरचना में अनिसोट्रॉपी को दूसरी श्रेणी के फैब्रिक टेंसर के रूप में गणितीय रूप से व्यक्त किया जा सकता है। यहां, हम हिम के लिए एक स्ट्रेस - स्ट्रेन संबंध प्रस्तुत करते हैं जो कि फैब्रिक टेंसर के माध्यम से इसमें निर्मित सूक्ष्म संरचना पर निर्भर है।

हिम के फैब्रिक आधारित इलास्टिक और स्ट्रेंथ गुणों के संबंध हाल ही में सफलतापूर्वक अनुमान लगाए गए हैं। इससे प्रेरित होकर, हम हिम के लिए एक फैब्रिक-आधारित मैक्रोस्कोपिक इलास्टो-प्लास्टिक संवैधानिक कानून का प्रस्ताव करते हैं, जिसका उपयोग हिमस्खलन की शुरुआत का अध्ययन करने के लिए किया जा सकता है। इस प्रक्रिया में आइसोट्रोपिक हार्डनिंग/सॉफ्टनिंग के प्रावधान के साथ फैब्रिक टेंसर और घनत्व-निर्भर यील्ड सतह का उपयोग किया जाता है। प्रारंभिक यील्ड से परे, यील्ड फलन तब तक बढ़ता है जब तक हिम की स्ट्रेंथ तक नहीं पहुंच जाती और फिर नरम हो जाती है। चूंकि बर्फ टेंशन-कम्प्रेशन व्यवहार विषमता प्रदर्शित करता है, एक टुकड़ा-वार द्विघात यील्ड फ़ंक्शन का उपयोग किया जाता है ।

अध्ययन विभिन्न घनत्व वाले हिम के नमूनों पर किया जाता है जिसमें गोल ग्रेन (आरजी), फेशियल क्रिस्टल (एफसी), और डेप्थ होर (डीएच) हिम क्लास शामिल हैं। प्रत्येक हिम के नमूने के लिए मीन इंटरसेप्ट लेंथ आधारित फैब्रिक टेंसर (एम्आईएल फैब्रिक) एक्स-रे माइक्रो-कंप्यूटेड टोमोग्राफिक (माइक्रो-सीटी) डेटा से निर्धारित होता है । प्रत्येक हिम के नमूने के ३डी माइक्रो-एफई मॉडल के निर्माण के लिए माइक्रो-सीटी डेटा कार्यरत है । प्रत्येक नमूने के समरूप स्ट्रेस-स्ट्रेन प्रतिक्रिया और यांत्रिक गुणों को उनके ३डी माइक्रो-एफई मॉडल से अलग-अलग सीमा स्थितियों के अधीन निर्धारित किया जाता है। फैब्रिक-इलास्टिक और

फैब्रिक-स्ट्रेंथ मॉडल के अज्ञात मापदंडों का मूल्यांकन हिम के नमूनों के समरूप इलास्टिक, स्ट्रेंथ और फैब्रिक डेटा से किया जाता है। आइसोट्रोपिक हार्डनिंग/सॉफ्टनिंग फंक्शन की निरंतरता को हिम नमूनों के समरूप स्ट्रेस-स्ट्रेन और संचित प्लास्टिक स्ट्रेन डेटा से निर्धारित किया जाता है।

३डी में फैब्रिक-आधारित विफलता सतह, द्विअक्षीय नार्मल-नार्मल और नार्मल-शियर का निर्माण किया जाता है, जो टेंशन-कम्प्रेशन विषमता को दर्शाता है। फैब्रिक-आधारित तनाव-तनाव प्रतिक्रिया की भविष्यवाणी करने के लिए मैक्रोस्कोपिक संवैधानिक कानून को एफई कोड (वीयूएम्एट) के रूप में लागू किया गया है। उच्च घनत्व वाले आरजी, एफसी, डीएच, और एक अक्षीय और सीमित संपीड़न में कम-घनत्व एफसी हिम के लिए मैक्रोस्कोपिक संवैधानिक कानून और माइक्रो-एफई आधारित स्ट्रेस-स्ट्रेन प्रतिक्रिया की तुलना की जाती है। फैब्रिक और माइक्रो-एफई डेटा ने अच्छा मेल दिखाया।

मैक्रोस्कोपिक संवैधानिक कानून मजबूत और कमजोर परतों वाले स्तरित हिमखंड पर लागू होता है। हिमखंड पर हाल के अध्ययन, जैसे स्कीयर और गुरुत्वाकर्षण भार के अधीन, प्रोपेगेशन सॉ टेस्ट (पीएसटी), की जांच फैब्रिक -आधारित मॉडल से की जाती है। एक कमजोर परत में स्ट्रेसवितरण, विभिन्न स्लैब और कमजोर परत की मोटाई पर, और पीएसटी आउटपुट अर्थात महत्वपूर्ण दरार लंबाई, स्लैब विफलता, फ्रैक्चर प्रसार गति, और फ्रैक्चर प्रसार दूरी की भविष्यवाणी मैक्रोस्कोपिक कानून-आधारित जांच से की जाती है और रिपोर्ट किए गए डेटा के साथ तुलना की जाती है। रिपोर्ट किया गया और मैक्रोस्कोपिक कानून-आधारित डेटा उचित रूप से मेल खाता है। स्तरित हिमखंड में यांत्रिक व्यवहार पर फैब्रिक के प्रभाव को दिखाया गया है और प्रारंभिक फैब्रिक आधारित विश्लेषण के साथ तुलना की गई है।

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LIST OF SYMBOLS

μ-FE	Micro-Finite Element
μCT	X-ray micro-computed tomography
$\boldsymbol{n_i}$	Orientation vector in ith direction
MIL, L	Mean intercept length
h_k	Length of the kth intercept segment
I	Number of all the intercept segments
H	Second rank material tensor
x_i, y_i, z_i	Direction cosines
M	Fabric tensor
m_i	Eigenvalue of **M**
$\boldsymbol{m_i}$	Eigenvectors of **M**
$\mathbf{M_i}$	Second order eigenvector tensor
SLD	Star length distribution
SVD	Star volume distribution
RG	Rounded Grains
FC	Faceted Crystals
DH	Depth Hoar
FCsf	Near surface faceted particles
p_0	compressive strength
M	slope of critical state line
β	ratio of tensile and compressive strength

p	Hydrostatic stress
q	von-Mises stress
$\tilde{\boldsymbol{s}}$	Deviatoric stress
r	Third invariant of deviatoric stress
ϕ	Internal friction angle
c, d	Cohesion
K	Triaxiality ratio
κ_{ice}	Accumulated plastic strain of ice
$\tilde{\boldsymbol{\varepsilon}}_{\boldsymbol{ice}}$	Total strain of ice
$\tilde{\boldsymbol{\varepsilon}}_{\boldsymbol{e}_{\boldsymbol{ice}}}$	Elastic strain tensor of ice
$\tilde{\boldsymbol{\varepsilon}}_{\boldsymbol{p}_{\boldsymbol{ice}}}$	Plastic strain tensor of ice
H_{ice}	Hardening modulus of ice
$\sigma_Y(\kappa)_{ice}$	Yield strength of ice
$(\sigma_{y0})_{ice}$	Initial yield strength in compression
$\overline{\omega}$	Direction independent damage in ice
$\boldsymbol{\sigma}_{\boldsymbol{ice}}$	Nominal stress in ice
$\mathbb{C}_{\boldsymbol{o}}$	Initial stiffness of ice
L	Characteristic length of each element
δ	Plastic displacement after damage initiation
δ_{pl}	Plastic displacement corresponding to the failure
G_f	Fracture energy release rate
$\overline{\sigma}$	Effective stress

ρ_{ice}	Density of ice
ρ_s	Density of snow
E	Young's Modulus of ice
ν	Poisson's ratio of ice
κ_0	Initial accumulated plastic strain
RVE	Representative Volume Element
$\bar{\boldsymbol{\sigma}}$	homogenized stress
$\bar{\boldsymbol{\varepsilon}}^{\boldsymbol{e}}$	Homogenized elastic strain
V	Total volume of the cube
$\boldsymbol{\sigma}$	Stress in each element
$\hat{\sigma}$	Normalised strength
$\mathbb{E}$	Elastic compliance tensor
$\mathbb{S}$	Orthotropic stiffness tensor
ρ	Ice volume fraction
ϵ_i	Young’s moduli of snow
ν_{ij}	Poisson’s ratio of snow
μ_{ij}	Shear moduli of snow
ϵ_0	Elastic modulus of pore less snow
ν_0	Poisson’s ratio of pore less snow
μ_0	Shear modulus of pore less snow
k	Exponent of ρ in elastic regime
l	Exponent of fabric eigen value in elastic regime

p	Exponent of ρ in post-yield regime
q	Exponent of fabric eigen value in post-yield regime
$\mathrm{E_{eff}}$	Effective isotropic Young's modulus
ν_{eff}	Effective Poisson’s ratio
$\mathrm{k_{eff}}$	Effective isotropic bulk
$\mathrm{G_{eff}}$	Effective shear moduli
$\mathrm{S_{ij}}$	Components of $\mathbb{S}$ tensor
$\mathrm{E_{ij}}$	Components of $\mathbb{E}$ tensor
κ	Accumulated plastic strain
$r(\kappa)$	Hardening/softening function
$\mathbb{F}^{\pm}$	Fourth-order strength tensor in tension (+) and compression (-)
$\sigma_{ii}^{\pm}$	Uniaxial tension (+) and compressive (-) strengths in I direction
τ_{ij}	Shear strengths in the principal plane of index i, j=1,2,3, i ≠ j
$\chi_{ij}^{\pm}$	Interaction terms in the principal plane of index i, j=1,2,3, i ≠ j
$\boldsymbol{\sigma}$	Total stress
h	Hyperplane
$\dot{\boldsymbol{\varepsilon}}^{\boldsymbol{p}}$	Plastic strain rate
$\dot{\lambda}$	Plastic multiplier
$\dot{\kappa}$	Accumulated plastic strain
y_r	Strength ratio
$g(\kappa)$	Piecewise polynomials (function of κ)
$\boldsymbol{d\sigma}$	Stress increment

$\boldsymbol{d\varepsilon}$	Strain increment
$\boldsymbol{d\varepsilon^p}$	Plastic strain increment
ET	Equilibrium temperature
TG	Temperature gradient
MF	Melt-Freeze
k_V	Voigt bound for isotropic bulk modulus
k_R	Reuss bound for isotropic bulk modulus
G_V	Voigt bound for isotropic shear modulus
G_R	Reuss bound for isotropic shear modulus
$\otimes$	Double tensor product, $\boldsymbol{A} \otimes \boldsymbol{B} = A_{ij}B_{kl}$
$\overline{\underline{\otimes}}$	Double tensor product, $\boldsymbol{A}\, \overline{\underline{\otimes}}\, \boldsymbol{B} = \frac{1}{2}\left(A_{ik}B_{jl} + A_{il}B_{jk}\right)$
DEM	Dicrete element method
FEA	Finite element analysis
DP	Drucker Prager yield criteria
σ_n	Normal stress
τ	Shear stress
MC	Mohr’s Coulomb criteria
MCC	Mohr’s Coulomb cap model
PST	Propagation Saw Test
g	Acceleration due to gravity
Δy	Total vertical collapse of weak layer
H	Slab thickness

φ	Slope angle
FPS	Fracture propagation speed
FPD	Fracture propagation distance
KE	Kinetic energy
h_w	Thickness of weak layer
h_s	Thickness of slab
l	Length of the snowpack
p_t	Exponent of ρ in tension state
p_c	Exponent of ρ in compressive state
p_s	Exponent of ρ in shear state.
MPM	Material Point Method
b	Width of the snowpack
σ_N	Normalised strength
J_2	Second invariant of deviatoric stress