THEORETICAL STUDY OF CARDIOVASCULAR FLOWS IN SMALL AND LARGE BLOOD VESSELS

BY
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Dedicated to my Parents ......

CERTIFICATE

This is to certify that the thesis entitled "**THEORETICAL STUDY OF CARDIOVASCULAR FLOWS IN SMALL AND LARGE BLOOD VESSELS**" which is being submitted by Mr. Ranjan Kumar Dash to the **INDIAN INSTITUTE OF TECHNOLOGY, DELHI** for the award of the degree of **DOCTOR OF PHILOSOPHY** in Mathematics is a record of bonafide research work carried out by him under our guidance and supervision.

The thesis has reached the standard fulfilling the requirements for the award of the degree. The results obtained in this thesis have not been submitted to any other university or institution for the award of any other degree or diploma.

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ABSTRACT

This thesis attempts to understand some of the clinically relevant aspects of blood flow in the cardiovascular system through mathematical modelling. It consists of six chapters; the first chapter being an introductory one giving a brief description of modelling in the cardiovascular system. The rest of the chapters are divided into two parts. **PART I** (chapters II to IV) deals with flow/dispersion in small blood vessels (diameters in the range 0.1 mm to 1 mm) with blood modelled appropriately as a Casson fluid - a rheological fluid model which takes into account the effect of yield stress of the fluid. **PART II** (chapters V and VI) deals with flows in large blood vessels (diameters exceeding 1 mm). Since the focus in this part is more on the effects of stenosis and curvature on blood flow dynamics, the Newtonian fluid model for blood is preferred.

CHAPTER II deals with the study of changed flow pattern and estimation of increased frictional resistance in a narrow artery in the presence of a catheter. Both the cases of steady and pulsatile flow situations are studied. The pulsatile flow is studied under the action of a periodic pressure gradient with small inertial effects. The resulting quasi-steady non-linear coupled implicit system of differential equations governing the flow are solved by employing a perturbation series analysis about the Womersley frequency parameter α which is small. The effect of pulsatility, catheter radius, and yield stress of blood on the location of yield planes, velocity field in the plug and shear flow regions, flow rate, and wall shear stress are investigated.

CHAPTER III deals with the flow of a Casson fluid in a homogeneous porous medium bounded by a circular tube. The objective is to model the blood flow in artificial devices which use highly selective membranes made of synthetic porous surfaces and extracellular matrices. Both the cases of constant and variable permeability are studied. The Brinkman model, which takes care of the boundary layer nature of the flow field, is employed to account for the Darcy resistance offered by the porous medium. The non-

linear coupled implicit system of differential equations governing the steady flow is first transformed into two simultaneous integral equations which are then solved numerically using an iterative technique. Analytical solution is obtained for a Newtonian fluid for the case of constant permeability, and the numerical solution is verified with the analytical solution. The effects of yield stress of the fluid and permeability of the porous medium on shear stress and velocity distributions, plug flow radius and flow rate are examined.

CHAPTER IV deals with the dispersion of a bolus of solute in the unidirectional flow of a Casson fluid in a conduit (pipe/channel). The study is relevant for understanding many physiological processes which involve the introduction of a quantity of solute into the blood stream and measuring its concentration at some downstream points (e.g., (i) dye or indicator dilution technique to measure cardiac output and (ii) dispersion of drugs/nutrients in the circulatory system). The generalized dispersion model of Gill and Sankarasubramanian (1970) is employed to study the dispersion process. This way, the effective diffusion coefficient, which describes the whole dispersion process in terms of a simple diffusion process, is obtained as a function of time. It also depends on the specific rheological parameter (i.e., the yield stress) of the fluid. The effect of yield stress of the fluid on the dispersion process is examined.

CHAPTER V deals with the fluid mechanics of blood flow in a catheterized curved artery with stenosis. The wall of the stenosed artery is considered as sinusoidally varying ($\eta(z) = 1 - \delta_1 \sin\pi z$, where δ_1 is the maximum height of the stenosis, called stenotic parameter). Since the governing equations of motion are non-linear in nature, an approximate analytical solution is attempted through a double series perturbation analysis about the curvature parameter $\epsilon = a/b$ and the geometric parameter $\delta = a/L$ (where a is the radius and b is the radius of curvature of the curved artery, and L is the length of the stenosis) which are assumed to be small. From this analysis, the flow characteristics in a catheterized curved artery of uniform cross-section are obtained as a special case $\delta_1 = 0$. The effect of catheterization on various physiologically important flow characteristics - i.e., pressure drop, impedance and wall shear stress - is studied for different values of the Reynolds number Re of the flow. The results are used to obtain the estimates for increased pressure drop (and hence, impedance) and wall shear stress across a coronary artery stenosis during catheterization. In addition, many interesting fluid mechanical phenomena - i.e., the modification of secondary streamlines due to the combined action of stenosis and curvature, and formation of increased number of secondary vortices due to catheterization - are brought out in this investigation.

Although the analysis in chapter V is based on small values of ϵ, δ and δ_1, the convergence of the perturbation series is found to depend mainly on the values of the Reynolds number Re and the ratio k of catheter size to vessel size. The range of Re for which the series converges depends on the value of k and vice-versa. For larger values of k, the series converges for relatively smaller values of Re. The limitations of the analytic approach of chapter V are overcome in the numerical approach of chapter VI.

In **CHAPTER VI**, an attempt has been made to obtain the flow characteristics in a catheterized curved artery with stenosis for larger values of Reynolds number Re through numerical methods. The three-dimensional non-linear elliptic partial differential equations governing the flow are simplified by (i) scaling the velocity components appropriately so as to make the centrifugal force terms to be of the same order of magnitude as the viscous and inertial terms and (ii) using the small curvature (i.e., $\epsilon \ll 1$) approximation for mild stenosis (i.e., $\delta \ll 1$ and $\delta_1 \ll 1$). This way, the Dean number D - the dynamical similarity parameter for curved tube flows - which is defined in terms of the Reynolds number Re and curvature parameter ϵ by $D = 4\,(2\epsilon)^{1/2}$ Re appears naturally in the simplified equations. The effects of curvature and stenosis are taken care through Dean number D and the boundary condition at the arterial wall respectively. The simplified equations of motion are solved numerically using the finite difference schemes described by Collins and Dennis (1975). The effects of Dean number D (or equivalently, Reynolds number Re) and catheter radius k on various physiologically important flow characteristics - i.e., flow rate, pressure drop, impedance, wall shear stress and the change in flow pattern - are investigated.

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