QUANTUM MECHANICAL TUNNELLING

by

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CERTIFICATE

This is to certify that the dissertation entitled "Quantum Mechanical Tunnelling". Which is being submitted by Mr. Amitabh Ghosh to Indian Institute of Technology, Delhi is a record of bonafide research work carried out by him under my guidance and supervision. He has also pursued the prescribed course of research.

I am satisfied that the thesis presented by him is worthy of consideration for the award of the degree of Doctor of philosophy. The results contained in it have not been transmitted in part or in full to any University or Institute for any award of degree or diploma.

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ABSTRACT

The origin of the idea of quantum mechanical tunnelling and its subsequent experimental verification provide one of the examples of early triumphs of wave mechanics. According to conventional quantum theoretical predictions a particle in one allowed region of space, owing to its wave nature has a non-zero probability of penetrating through a classically forbidden zone, called a potential barrier, into a second classically allowed region of space. The conventional way of analysing the above problem is to solve the Schrodinger's time-independent wave equation inside the barrier. Such an approach is based upon the assumption that the matter is capable of retaining its duality in the barrier region where one can regard its motion to be taking place as if through a conservative field of force so that it is always possible to take account of its energy at every point of its motion by virtue of the principle of conservation of energy. Although such an approach had a number of early successes in explaining various phenomena namely electron emission from cold metals, α -decay from heavy nuclei etc, it had its record of failures too due to its inabilities in explaining Esaki and Josephson effects adequately as well as in accounting for the energy distribution effect of electrons observed in field emission from cold metals. The aforesaid time-independent or stationary state approach also suffers from certain conceptual problems. By a potential barrier one understands a region of space where a particle possessing an energy smaller than its height can never exist. Therefore, the question of finite probability density of particles or of their exhibiting duality during tunnelling should not arise. But conventionally a potential

barrier is considered to be translucent to an electron gas which is incident upon it inspite of the fact that the energy of each particle happens to be smaller than the height of the barrier.

Attempts have been made to improve upon this concept by proposing one time-dependent perturbation or the transfer Hamiltonian formalism of tunnelling. But here also one presumes the existence of quantized states even inside the barrier which are characterized by decaying and growing wave functions. The phenomenon of tunnelling is supposed to be caused due to transition of particles from the former to the latter by an unidentified perturbing term in the barrier Hamiltonian. Since such a perturbation has to be very small this approach in the limit is also found to be identical with the time-independent description of tunnelling discussed earlier. Both models have, however, failed to provide satisfactory definitions and calculations of tunnelling time and tunnelling current density.

From this brief resume it should now be apparent that the very conceptual foundation of tunnelling theories demand a major as well as a thorough scrutiny. The very purpose of this thesis is to present such discussions in a coherent fashion and thereafter to develop them appropriately so that they are not only able to account for all the existing results observed so far in this area but are able to predict a few new ones also. The problem of tunnelling has been analysed in this thesis from the point of view of quantum measurement and observation. The latter, infact, lies at the very root of the wave-particle dualism manifested by matter. According to it, a matter wave after interacting with a classical object for a certain length of time acquires the shape of a wave packet which determines its subsequent particle like behaviour. In the case of tunnelling, the barrier is taken to function as a classical object with which the incident matter wave interacts to generate a wave packet at the transmitted end. Hence the interaction time is the tunnelling time. The characteristics of the generated wave packets are, however, determined by heights and widths of the potential barriers in a manner to be discussed in the thesis. Since a wave-packet results due to the interaction of the matter waves with the classical object, the process of interaction has been termed "quantum measurement". Also, since the presence of a classical object is absolutely necessary for such a measurement to be carried out, the former has appropriately been referred to as the "apparatus".

During the interaction of the matter wave with the classical object or the barrier, it behaves neither like a wave nor as a particle. Hence, during the quantum measurement process or tunnelling it is not possible to assign to it its true energy. Once the interaction is over and a wave packet is nucleated, the latter starts behaving as a group of particles or a group of waves. The said wave packet has its characteristic momentum and energy uncertainties about the original momentum and energy possessed by the incident particles. These uncertainties are determined by the width and the height of the barrier as specified by Heisenberg's uncertainty relations. This fact clearly explains why the particle probability density should be observed distributed in energy about its incident value after tunnelling, as observed experimentally and not restricted to be field emitted only at the incident energy, as formerly believed.

But the tunnel current density across a solid-state junction has always been observed to be emitted only at the incident energy. Such a current is usually produced due to the incidence of incoherent stream of particles upon the barrier which interact with it for a considerable length of time which is very large compared to the single particle tunnelling time. Since such interaction time is very large, the energy uncertainty of the wave packet that results due to interaction tends to be vanishingly small as predicted by Heisenberg's uncertainty relation. This explains why the tunnelling current produced by an incoherent stream of particles should appear to be emitted at the incident energy only.

In the chapter I of our thesis we have summarized the conventional theories of tunnelling essentially with a view to bring out their deficiencies. The chapter II in it has next been devoted fully to the development of the new theory on tunnelling based on the principle of quantum measurement and observation mentioned above. The case of resonant tunnelling has also been discussed therein from our new approach. The concept of indirect tunnelling has been developed afresh and its mathematical formalism is presented in detail in chapter III. Chapter IV, on the other hand, presents magneto-tunnelling effect from our new point of view. A suggestion for a possible experimental measurement of tunnelling time has been made in it so that our prediction could be confirmed. Chapter V at the end summarizes the thesis and brings out its important conclusions. A few possible future lines of research have also been suggested therein.

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