METAPILE: RESONATOR IMPREGNATED PILE FOUNDATION FOR VIBRATION MITIGATION

RISHAB DAS

DEPARTMENT OF CIVIL ENGINEERING

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Metapile: Resonator impregnated pile foundation for vibration mitigation

by

Rishab Das

Department of Civil Engineering

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CERTIFICATE

This is to certify that the thesis titled **Metapile: Resonator impregnated pile foundation for vibration mitigation**, submitted by **Mr. Rishab Das**, to the Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by him under our supervision and guidance. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Bappaditya Manna
Professor
Dept. of Civil Engineering
IIT Delhi, 110016

Dr. Arnab Banerjee
Assistant Professor
Dept. of Civil Engineering
IIT Delhi, 110016

New Delhi
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(Rishab Das)

ABSTRACT

The current research presents a comprehensive investigation into the dynamic response of monopile foundations embedded in homogeneous and layered soil-rock systems under horizontal dynamic loads. The monopile is modeled using visco-elastic springs distributed along its length, representing its interaction with the elastic half-space. The frequency-dependent stiffness and damping of the monopile in various soil-rock conditions are derived through spectral element formulation, demonstrating their variation with frequency. Findings reveal that the depth of rock-socketed monopiles significantly enhances system stiffness, with horizontal amplitudes decreasing and resonant amplitudes for rocking motion increasing as socketing depth grows. Furthermore, the study explores the influence of pile slenderness ratios on natural frequencies and peak amplitudes, providing insights into the dynamic characteristics of pile-soil-rock systems.

Building on this foundation, the research introduces the concept of the "metapile," a monopile integrated with periodically placed spring-mass resonators. A spectral element approach is employed to derive the dynamic stiffness matrix, which is condensed to obtain the system's impedance functions. The dynamic response of the metapile demonstrates reduced amplitudes within specific frequency ranges, owing to the resonators' ability to enhance stiffness near their resonant frequencies. Comparative analysis with monopiles augmented with equivalent lumped masses reveals the unique vibration attenuation capabilities of resonator-based systems, broadening the operational frequency range and improving vibration control.

The application of these principles to wind turbine systems is also investigated. Four configurations namely- conventional turbines, those with resonators in the tower, monopile, or both- are analyzed. Spectral element formulations calculate dynamic responses under harmonic excitations, with experimental validation using a scaled wind turbine model. Results highlight the effectiveness of resonators, particularly when integrated into both tower and monopile, in reducing transmittance near resonant frequencies and mitigating responses at higher excitation frequencies.

The study further examines energy dissipation in monopile systems modeled with viscoelastic springs, focusing on transient response decay. Using Bloch's theorem and dispersion relationships, a novel analytical method estimates dissipation based on damping ratios over the Brillouin zone. Validation with numerical models confirms the efficacy of resonators in enhancing dissipation, termed "metadamping." Parametric studies reveal the impact of resonator characteristics on dissipation, offering a framework for designing resilient pile foundations.

Finally, the research investigates the dynamic interaction of closely spaced pile groups under horizontal vibrations. A detailed framework accounts for primary and secondary wave propagation, soil property variations, and weak zone effects. Results indicate that increasing soil stiffness ratios and boundary zone thickness significantly influences stiffness and damping, providing design guidance for optimizing pile group performance in dynamic environments.

This thesis contributes novel insights into the dynamic behavior and control of monopile foundations, offering practical solutions for improving stability and vibration mitigation in onshore infrastructure systems.

सार

वर्तमान शोध क्षैतिज गतिशील भार के तहत सजातीय और स्तरित मिट्टी-चट्टान प्रणालियों में एम्बेडेड मोनोपाइल नींव की गतिशील प्रतिक्रिया में एक व्यापक जांच प्रस्तुत करता है। मोनोपाइल को इसकी लंबाई के साथ वितरित विस्को-लोचदार स्प्रिंग्स का उपयोग करके मॉडल किया गया है, जो लोचदार अर्ध-स्थान के साथ इसकी बातचीत का प्रतिनिधित्व करता है। विभिन्न मिट्टी-चट्टान स्थितियों में मोनोपाइल की आवृत्ति-निर्भर कठोरता और भिगोना स्पेक्ट्रल तत्व निर्माण के माध्यम से प्राप्त किया जाता है, जो आवृत्ति के साथ उनकी भिन्नता को प्रदर्शित करता है। निष्कर्ष बताते हैं कि रॉक-सॉकेट किए गए मोनोपाइल की गहराई सिस्टम की कठोरता को काफी हद तक बढ़ाती है, जिसमें क्षैतिज आयाम कम हो जाते हैं और रॉकिंग गति के लिए अनुनाद आयाम बढ़ते हैं क्योंकि सॉकेटिंग गहराई बढ़ती है। इसके अलावा, अध्ययन प्राकृतिक आवृत्तियों और शिखर आयामों पर पाइल स्लेंडरनेस अनुपात के प्रभाव की खोज करता है, जो पाइल-मिट्टी-चट्टान प्रणालियों की गतिशील विशेषताओं में अंतर्दृष्टि प्रदान करता है। इस आधार पर निर्माण करते हुए, शोध "मेटापाइल" की अवधारणा को प्रस्तुत करता है, जो समय-समय पर रखे गए स्प्रिंग-मास रेज़ोनेटर के साथ एकीकृत एक मोनोपाइल है। गतिशील कठोरता मैट्रिक्स प्राप्त करने के लिए एक वर्णक्रमीय तत्व दृष्टिकोण का उपयोग किया जाता है, जिसे सिस्टम के प्रतिबाधा कार्यों को प्राप्त करने के लिए संघनित किया जाता है। मेटापाइल की गतिशील प्रतिक्रिया विशिष्ट आवृत्ति सीमाओं के भीतर कम आयाम प्रदर्शित करती है, जो अनुनादकों की अपनी अनुनाद आवृत्तियों के पास कठोरता को बढ़ाने की क्षमता के कारण है। समतुल्य ढेरदार द्रव्यमानों के साथ संवर्धित मोनोपाइल्स के साथ तुलनात्मक विश्लेषण अनुनादक-आधारित प्रणालियों की अद्वितीय कंपन क्षीणन क्षमताओं को प्रकट करता है, परिचालन आवृत्ति सीमा को व्यापक बनाता है और कंपन नियंत्रण में सुधार करता है।

पवन टरबाइन प्रणालियों में इन सिद्धांतों के अनुप्रयोग की भी जांच की जाती है। चार विन्यास अर्थात्- पारंपरिक टर्बाइन, टॉवर में अनुनादक वाले, मोनोपाइल, या दोनों- का विश्लेषण किया जाता है। स्पेक्ट्रल तत्व सूत्र एक स्केल्ड पवन टरबाइन मॉडल का उपयोग करके प्रयोगात्मक सत्यापन के साथ, हार्मोनिक उत्तेजनाओं के तहत गतिशील प्रतिक्रियाओं की गणना करते हैं। परिणाम अनुनादकों की प्रभावशीलता को उजागर करते हैं, विशेष रूप से जब टॉवर और मोनोपाइल दोनों में एकीकृत होते हैं, अनुनाद आवृत्तियों के पास संप्रेषण को कम करने और उच्च उत्तेजना आवृत्तियों पर प्रतिक्रियाओं को कम करने में। अध्ययन में विस्को-इलास्टिक स्प्रिंग्स के साथ मॉडल किए गए मोनोपाइल सिस्टम में ऊर्जा अपव्यय की आगे की जांच की गई है, जो क्षणिक प्रतिक्रिया क्षय पर ध्यान केंद्रित करता है। ब्लोच के प्रमेय और फैलाव संबंधों का उपयोग करते हुए, एक नई विश्लेषणात्मक विधि ब्रिलौइन क्षेत्र पर भिगोना अनुपात के आधार पर अपव्यय का अनुमान लगाती है। संख्यात्मक मॉडल के साथ सत्यापन अपव्यय को बढ़ाने में अनुनादकों की प्रभावकारिता की पुष्टि करता है, जिसे "मेटाडैम्पिंग" कहा जाता है। पैरामीट्रिक अध्ययन अपव्यय पर अनुनादक विशेषताओं के प्रभाव को प्रकट करते हैं, जो लचीले ढेर नींव को डिजाइन करने के लिए एक रूपरेखा प्रदान करते हैं।

अंत में, शोध क्षैतिज कंपन के तहत निकट दूरी वाले ढेर समूहों की गतिशील बातचीत की जांच करता है। एक विस्तृत रूपरेखा प्राथमिक और द्वितीयक तरंग प्रसार, मिट्टी की संपत्ति भिन्नता और कमजोर क्षेत्र प्रभावों के लिए जिम्मेदार है। परिणाम संकेत देते हैं कि मिट्टी की कठोरता अनुपात और सीमा क्षेत्र की मोटाई में वृद्धि कठोरता और भिगोना को महत्वपूर्ण रूप से प्रभावित करती है, जो गतिशील वातावरण में ढेर समूह के प्रदर्शन को अनुकूलित करने के लिए डिजाइन मार्गदर्शन प्रदान करती है।

यह शोध प्रबंध मोनोपाइल नींव के गतिशील व्यवहार और नियंत्रण के बारे में नवीन अंतर्दृष्टि प्रदान करता है, तथा तटवर्ती अवसंरचना प्रणालियों में स्थिरता और कंपन शमन में सुधार के लिए व्यावहारिक समाधान प्रस्तुत करता है।

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List of symbols

Roman Symbols

a	Dimensionless frequency parameter
$C.G$	Center of gravity
c_u, c_v	Constants of the differential equation of U_{cb} and V_{cb}
c_s	Damping component of the soil
C_p	Equivalent damping component of the pile-soil system
dz	length of a small section
D	Diameter of the cross-sections
D'	nodal displacement field of a single pile element
D'_c	Displacement field of a single curve section
D^*	Global displacement field of a single curve section
d_p	Pile outer diameter
dz	Height of pile element
E_p	Young's modulus of the pile
F_H	Nodal force matrix of a pile element
F^*	Global force matrix of a curve element
F_{mp}	Nodal force matrix of a resonator infused pile element
G	Shear modulus of the soil
G_i	Shear modulus of the weak zone around the pile
G_o	Shear modulus of the outer zone of the soil
$\Im$	Imaginary part of a function
$I_{0,1}$	Bessel function of the first kind of order 0 or 1
I_p	Area moment of inertia of the pile
I'_p	Polar area moment of area
I_ψ	Mass moment of inertia of the block foundation

k_s	Spring stiffness component of the soil
k_t	Timoshenko constant
K_{ui}	Impedance function within the boundary zone of soil
K_{u0}	impedance function of the outer medium of soil
K_p	Equivalent stiffness of the pile-soil system
$K_{0,1}$	Bessel function of the second kind of order 0 or 1
$\mathbf{K}_{rr}$	Stiffness submatrix corresponding to the retained degrees of freedom
$\mathbf{K}_{rc}$	Submatrix containing the rest of the row elements of the stiffness matrix
$\mathbf{K}_{cr}$	Submatrix containing the rest of the column elements of the stiffness matrix
$\mathbf{K}_{cc}$	Stiffness submatrix corresponding to the degrees of freedom that are to be condensed
K_{gxx}	Impedance of the pile group
L_1 , L_2 , L_3 , L_4	Invariants of the polynomial eqution for the in-plane/ out-of-plane displacement of curve beam
L_p	Length of the pile
$\frac{L}{r}$	Slenderness ratio of the pile
lm	Diameter of the cylindrical rigid mass of the resonator
M_f	Mass of the foundation block
$m_o(\phi, t)$	Out-of-plane bending moment about the radial axis
m_r	Mass of the resonator
$M(i)$	Moment at the i^{th} location
M	In-plane bending moment
M_p	Equivalent mass of the pile-soil system
N	In-plane normal force acting on a cross-section
$N_p(\lambda z)$	Shape function of the pile
$p(\phi, t)$	Out-of-plane shearing force
Q	In-plane shearing force acting on a cross-section
r	Radius of pile
ρ_p	Density of the pile
S	Spacing between two piles
$S_{mp}(\lambda)$	Spectral element matrix of the resonator infused pile
$S_p(\lambda)$	Spectral element matrix of the pile
$S'_p(\lambda)$	Spectral element matrix of the resonator infused pile

$S_t(\lambda)$	Spectral element matrix of the turbine tower
$S'_t(\lambda)$	Spectral element matrix of the resonator infused turbine tower
S_{u1}	Real part of complex Bessel function of soil
$\Re$	Real part of a function
$t^*(\phi, t)$	Out-of-plane bending about the longitudinal axis
t	Time parameter
t_s	boundary zone thickness of soil
t_p	Pile thickness
t_t	Curved beam thickness
T_P	Transfer matrix of the pile placed in soil
T_r	Transmittance in logarithmic scale
T_s	The maximum time frame response of the pile
T_t	Transformation matrix
u_c	In-plane shear deformation
$u(z, t)$	Horizontal deflection of pile at height z
u_h	Horizontal displacement at the pile head
u_g	Horizontal displacement at the pile bottom
U_{cb} , V_{cb}	In-plane displacement field of curve beam in frequency domain
U_i	Nodal displacement at the i^{th} node
v	Tangential displacement field of pile
v_c	In-plane axial deformation
$V(i)$	Shear force at the i^{th} location
V_c	Longitudinal wave velocity
V_s	Shear wave velocity
w	Out-of-plane shear deformation
w_r	Vertical displacement field of a single pile
Y_i	Amplitude of the displacement field of the infused resonators at the i^{th} node
z	Vertical field of the cylindrical coordinate system of pile-soil

Greek Symbols

α	Interaction factor within two piles
β	Angle of in-plane shear deformation
β_i	damping ratio of the weak zone of the soil

β_o	damping ratio of the outer zone of the soil
ϵ	Out-of-plane rotation about the radial axis
ϵ_s	Damping ratio
ϵ_{avg}	Mean measurement of the overall dissipation
γ	Complex exponential representation of a propagating wave in spacial domain
Γ_n	Modal participation factor of the n^{th} mode
λ	Roots of the differential equation of the pile-soil model
λ_n	Characteristic roots of the polynomial equation of V_{cb}
μ	Solutions of wavenumber
ν	PoissonâĂŹs ratio
Φ' and Ψ'	Potential functions of soil media
Φ_n	Eigen vectors of the n^{th} mode
ψ	In-plane bending slope
ψ_t	Out-of-plane rotation about the longitudinal axis
ρ	Mass density of the pile
σ_i	Normal stress of the i^{th} variable
τ_{ij}	Shear stress
θ	Angle between the deformed and undeformed neutral axis
θ_h	Rotation at the pile head
θ_i	Nodal rotation at the i^{th} node
Θ	Coefficient of overall dissipation
ω	Excitation frequency
ω_d	Damped frequency
ω_r	Natural frequency of the resonator
Ω	Ratio of the mass of the resonators with respect to the total mass of the pile
υ	order of the Bessel function of second kind
ϑ	Angular field of the cylindrical coordinate system of pile-soil
ζ_r	Damping ratio of the resonator