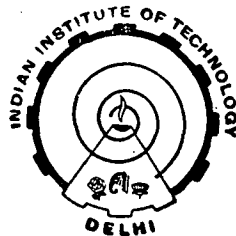


# **COUPLED FEBEM ANALYSIS OF UNDERGROUND OPENINGS**

by  
**RAJ BAL SINGH**

A THESIS SUBMITTED TO  
THE INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
FOR THE AWARD OF THE DEGREE OF  
**DOCTOR OF PHILOSOPHY**



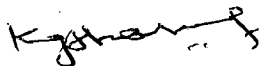
**Department of Civil Engineering**  
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March 1985

DEDICATED TO MY PARENTS  
AND UNCLE SHRI JAI SINGH

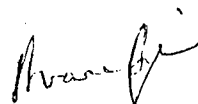
## CERTIFICATE

This is to certify that the thesis entitled, "COUPLED FEBEM ANALYSIS OF UNDERGROUND OPENINGS" being submitted by Mr. Raj Bal Singh to the Indian Institute of Technology, Delhi, for the award of degree of DOCTOR OF PHILOSOPHY is a record of the bonafide research work carried out by him. Mr. Raj Bal Singh has worked under our guidance for the submission of this thesis which, to our knowledge, has reached the requisite standard.

The thesis or any part thereof has not been submitted to any other University or Institution for the award of any degree or diploma.



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RAJ BAL SINGH

The modern technical aids of rock mechanics measurements and the use of computer programs are at the disposal of the engineer today to improve upon the design of underground openings even under difficult geotechnical conditions. The finite element method (FEM) and boundary element method (BEM) are two well established numerical methods used for the analysis of underground openings. Both the methods have got their advantages and disadvantages. In coupled finite element and boundary element method (FEBEM), the disadvantages of both the methods are eliminated by coupling them together. This is particularly advantageous in the case of underground opening in infinite medium and disturbance of material in the vicinity of the opening.

Keeping the above facts in view, the aim of this thesis has been to develop FEBEM for the analysis of underground openings; to study the effect of different factors viz., the fixation of interface boundary between finite element (FE) and boundary element (BE) regions, effect of Poisson's ratio, effect of stress ratio, comparison of different discretization schemes, and to compare the efficacy of this method with existing FEM by applying both the methods to analyse underground openings in elastic, elasto-plastic and two-layer rock media.

For this purpose, three computer programs have been developed using 4, 7 and 8-noded finite elements and 2 and 3-noded boundary elements. The computer program with 8-noded finite elements and 3-noded boundary elements has been extended for elasto-plastic and layered rock media analyses also. The excavation of the opening has been simulated in single stage for all the analyses.

A systematic investigation has been carried out for the fixation of interface boundary between FE and BE regions. Circular underground opening has been chosen for this purpose. Firstly, 4-noded linear finite elements have been used with 2-noded linear boundary elements. The interface boundary distance has been varied from two to eight times the radius of the opening. The Poisson's ratio has been varied from 0.0 to 0.49 for all the cases. The results are found to be dependent on the boundary distances and Poisson's ratios. The interface boundary distance at four times the radius of the opening has been found to be adequate. A similar study has also been carried out by FEM. In this case, the optimum boundary has been found to be at eight times the radius of the opening. It is found that FEBEM gives more accurate results than FEM when compared to the available closed form solutions. This is particularly so for higher values of Poisson's ratio and at points away from the excavation boundary.

Secondly, the effect of using higher order elements on the accuracy of the results and reduction in the number of equations and hence the computation time has been studied. For this purpose, 8-noded parabolic finite elements and 3-noded parabolic boundary elements have been used. The interface boundary distance between FE and BE regions has been varied together with Poisson's ratio. The optimum interface boundary distance has been found to be at four times the radius of the opening for FEBEM and eight times the radius of the opening for FEM. After fixing the boundary, the in-situ stress ratio ( $K_0$ ) has been varied as 0.0, 0.5 and 1.0. The stress ratio,  $K_0 = 1.0$  gives the least error and  $K_0 = 0.0$  shows the maximum error. Here also, FEBEM gives more accurate results than FEM. Further, FEBEM with 8-noded elements show practically negligible errors in the results as compared to 4-noded elements.

To take symmetry of the problem into consideration, the stiffness matrix of finite element region can easily be developed for a half or a quarter section of the opening. Two alternatives, viz., ALT1, which accounts for symmetry during the construction of boundary element stiffness matrix (BESM) and ALT2 which condenses the BESM developed for the full opening, have been used in FEBEM for boundary element region. ALT2 is preferred to ALT1 and the same is used in this thesis.

Further, a detailed investigation has been carried out to minimise the computation time without sacrificing the accuracy of the results. For this purpose, the circumferential and radial lines in the discretization have been varied to reduce the number of boundary elements to a minimum. Also, linear boundary elements have been coupled with 8-noded finite elements by using 7-noded finite elements near the interface. These results have been presented and discussed.

The major advantage of using FEBEM for the analyses of underground openings is that the change in material properties due to excavation near the opening can be easily taken into consideration. To illustrate this, an investigation has been carried out by varying the material properties in finite element region and using constant properties in boundary element region. The results of FEBEM analyses have been compared with FEM. It is found that for certain types of variation of material properties, there is significant difference in the results obtained by two methods.

The application of FEBEM has been extended for D-type shape of the opening. The location of interface boundary between FE and BE regions, effects of Poisson's ratio and stress ratio ( $\nu = 0.25, 0.49$  and  $K_0 = 0.5, 1.0$ ) have been considered. The interface boundary can be located at two times

the width of the opening. The results of FEBEM analyses have been compared by FEM fixing the boundary at four times the width of the opening.

The FEBEM has been developed to take into consideration the elasto-plastic behaviour of geological media. For this study, a field problem has been analysed using Mohr-Coulomb and no-tension failure criteria. The properties of Biotite Gneiss rock from Himalayan region have been taken with variation in stress ratio as 0.5, 1.0 and 1.5. The results of these analyses have been compared with those by FEM in terms of computation time, accuracy, number of iterations and effect of stress ratio.

The method is further extended to consider the layered rock medium. Two layers have been chosen for this study. The coupling of finite elements with boundary elements for analysing these cases has been presented. The elastic analyses of horizontal and inclined ( $60^{\circ}$ ) and elasto-plastic analysis of horizontal contacts of two rock layers have been considered. For all the cases, the lines of rock interfaces are assumed to be passing through the centre of the opening. The stress ratios of 0.5 and 1.0 and modulus ratios of two rocks equal to 1, 2 and 3 have been considered. The procedure is also discussed when the line of contact of two rock layers is not passing through the centre of the opening.

From all the above studies, it is concluded that FEBEM gives accurate results than those obtained by FEM.

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