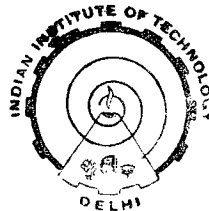


# MINIMUM REINFORCEMENT DESIGN OF REINFORCED CONCRETE SLABS

by

**MOHAMMAD ZAHOOR-UDDIN AHMAD**

THESIS SUBMITTED TO THE  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
FOR THE AWARD OF THE DEGREE OF  
**DOCTOR OF PHILOSOPHY**



Department of Civil Engineering  
**INDIAN INSTITUTE OF TECHNOLOGY, DELHI**  
**INDIA**

FEBRUARY, 1985

To my father  
Late Mr. MOHAMMAD ZAHIR-UDDIN AHMAD

CERTIFICATE

This is to certify that the thesis entitled "MINIMUM REINFORCEMENT DESIGN OF REINFORCED CONCRETE SLABS" being submitted by Mr. Mohammad Zahoor-uddin Ahmad to the Indian Institute of Technology, Delhi, in fulfilment of the requirements of the degree of Doctor of Philosophy, is a record of bonafide research work carried out by him under my guidance and supervision.

To the best of my knowledge, the thesis has reached the requisite standard. The matter presented in this thesis, has not been submitted in part or full to any other University or Institute for the award of any degree or diploma.



(Dr. T.K. Datta)  
Assistant Professor  
Department of Civil Engineering  
Indian Institute of Technology  
New Delhi-110016.

## ACKNOWLEDGEMENTS

I express my deep sense of gratitude to Dr. T.K. Datta for his inspiration and guidance at all stages of this work. His valuable suggestions and constant encouragement were of immense help in carrying out the present research work.

I am indebted to Prof. B.M. Ahuja for his valuable suggestions during the experimental work. I am also grateful to Prof. S. Krishnamoorthy and Mr. K.K. Nayar for having fruitful discussions and encouragement from time to time.

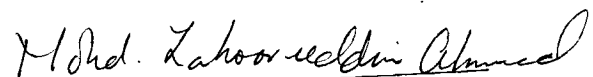
I also wish to acknowledge Mr. Arvind K. Jain and Ashok Gupta for their help in computational work. The cooperation extended by the staff of Computer Services Center, I.I.T., Delhi is also gratefully acknowledged.

My thanks are also due to Mr. Irshad Ahmad Khan, Mr. Mohammad Farid Husain and Mr. Abid Kazim Husain for their help at the final stages of this work. Thanks are also due to the staff of the Concrete and Structural Laboratory. The help rendered by Civil Engineering Work-shop is also gratefully acknowledged.

My special thanks are due to Mr. Dev Raj Joshi for typing the manuscript meticulously. I also express my thanks to Mr. Raj Veer Agrawal for preparing the tracings.

I am highly grateful to the Chairman, Civil Engineering Department and the Vice-Chancellor, Aligarh Muslim University for granting me leave in order to pursue this work.

Finally, I wish to express my special thanks to my wife and sons for their cooperation in pursuing my research work in Delhi.

  
(Mohammad Zahoor-Uddin Ahmad)

ABSTRACT

The present study is concerned with the minimum reinforcement design of rectangular slabs in which reinforcement is placed parallel to the edges of the slab. Two types of slabs namely, slabs supported all along their edges and typical interior panels of flat slab system are considered in the study. Three different types of boundary conditions are considered for the slabs supported all along their edges (i) all edges fixed, (ii) two longer edges fixed and other two simply supported, and (iii) two shorter edges fixed and other two simply supported. Three different minimum reinforcement solutions are presented for these slabs. The first solution obtains a minimum volume of straight reinforcement for rectangular slabs in which bottom reinforcement is curtailed near the edges. The solution is based on the theory of optimal plastic design for piece-wise constant reinforcement which uses Prager and Shield's optimality criterion. The second solution obtains the minimum volume of uncurtailed reinforcement based on the lower bound analysis (limit state) of the slab. The solution is obtained by choosing a statically admissible (twistless) moment field which minimizes the moment capacity volume of the slab. The minimum moment capacity volumes of the slabs obtained by this solution are nearly same as those obtained by Rozvany's static-kinematic solution for rectangular slabs with uncurtailed reinforcement. The third solution provides an economic reinforcement solution based on yield line theory. The work is an extension of Wood's economic reinforcement solution for simply supported square slab. The analysis considers curtailment of bottom reinforcement near the edges of the slab. The solution is achieved by obtaining a distribution of  $\frac{h}{\lambda}$ .

plastic moment capacities within the slab which will allow two modes of collapse to occur simultaneously. The results indicate that the moment capacity volumes of the slabs obtained by the present solution are about 5-35% less than those obtained by conventional yield line approach. The solution is, however, found to be less economic compared to the **first** solution mentioned earlier.

The enhancement in the load carrying capacities of the minimum reinforced slabs (laterally restrained at the edges) due to compressive membrane action is then studied both theoretically and experimentally. For the theoretical analysis, rigid plastic strip theory proposed by Park is used. The load carrying capacity of the slab is taken as the load corresponding to the central deflection of 0.5 times the overall depth of the slab. The results of the study show that the enhanced load carrying capacity of the minimum reinforced slabs are quite substantial and the serviceability behaviour of the slabs at the working load is satisfactory. Further, a parametric study shows that the coefficient of orthotropy (in the lower range  $\approx 0.25$  to 3.0) influences considerably the enhanced load carrying capacity of the slab due to compressive membrane action.

For flat slab panels, two minimum reinforcement solutions are presented. The first solution considers the slab panels on point supports. The minimum reinforcement solution is based on Prager and Shield's optimality criterion for piece-wise constant reinforcement. For the solution, two systems of

load transmission are considered; the first system transmits the load through strip action to the second system which carries the load to the point supports with a moment field in which twisting moment is present. The second solution considers the flat slab panels to be supported on rectangular columns of finite dimensions. For the analysis, a twistless moment field is assumed within the slab. The moment field is so selected that it minimizes the moment capacity volume of the slab.

Based on the minimum reinforcement solutions presented for rectangular slabs supported all along their edges and flat slab panels, tables for design moment coefficients have been prepared which can be readily used in practice.

## CONTENTS

	Page No.
ABSTRACT	i
LIST OF TABLES	iv
LIST OF FIGURES	vi
LIST OF PLATES	xii
CHAPTER I INTRODUCTION	1
CHAPTER II LITERATURE REVIEW	
2.1 INTRODUCTORY REMARK	11
2.2.1 Theories of Optimal Plastic Design	12
2.2.1.1 Single Load System	12
2.2.1.2 Multi-Component, Multi-Load System	15
2.2.2 Optimal Plastic Design of Flexural Continua	16
2.2.3 Optimal Design of Slabs with Variable Reinforcement	17
2.2.4 Lower Bound Solutions for Reinforced Concrete Slabs	21
2.2.5 Membrane Action in Reinforced Concrete Slabs	24
CHAPTER III MINIMUM VOLUME OF CURTAILED REINFORCEMENT IN RECTANGULAR SLABS	
NOMENCLATURE	30
3.1 INTRODUCTORY REMARK	33
3.1.1 Optimality Conditions	34
3.2 Reinforcement Arrangement	37
3.3 Assumptions	38
3.4 Rectangular Slab with All Edges Fixed	39
3.5 Rectangular Slab with Longer Edges Fixed and the Other Two Simply Supported	44
3.6 Rectangular Slab with Shorter Edges Fixed and the Other Two Simply Supported	47

3.7	Moment Capacity Volumes Obtained from the Displacement Field	49
3.8	Optimal Solution	50
3.9	Discussion of Results	51
3.10	Conclusions	55
	APPENDIX I	73
	APPENDIX II	76
	APPENDIX III	79
CHAPTER IV	MINIMUM WEIGHT DESIGN BASED ON LOWER BOUND SOLUTION	
	NOMENCLATURE	83
4.1	INTRODUCTORY REMARK	84
4.2	Reinforcement Arrangement	86
4.3	Assumptions	86
4.4	Moment Field For Lower Bound Solution	87
4.4.1	Slab with All Edges Fixed	88
4.4.2	Slab with Shorter Edges Fixed and Longer Edges Simply Supported	91
4.4.3	Slab with Longer Edges Fixed and Shorter Edges Simply Supported	93
4.5	Discussion of Results	95
4.6	Conclusions	97
CHAPTER V	ECONOMIC MINIMUM REINFORCEMENT SOLUTION BY YIELD LINE THEORY	
	NOMENCLATURE	114
5.1	INTRODUCTORY REMARK	116
5.2	Reinforcement Arrangement	117
5.3	Assumptions	118
5.4	Slab with all Edges Simply Supported	118
5.4.1	Work Equation for Mode I	119
5.4.2	Work Equation for Mode II	120
5.4.3	Evaluation of Yield Moment Capacities of the Slab	121
5.4.4	Minimization Problem	122

5.5	Slab with All Edges Fixed	123
5.5.1	Work Equation for Mode I	124
5.5.2	Work Equation for Mode II	125
5.5.3	Minimization of Moment Capacity Volumes	126
5.6	Slab with Two Opposite Edges Fixed and the Other Two Edges Simply Supported	127
5.7	Discussion of Results	127
5.8	Conclusions	130
CHAPTER VI	ENHANCED CAPACITY OF SLABS BASED ON OPTIMAL DESIGN	
	NOMENCLATURE	149
6.1	INTRODUCTORY REMARK	152
6.2	Compressive Membrane Action in the Post-Yield Stage	153
6.3	Assumptions	157
6.4	Development of the Method	158
6.4.1	Evaluation of Neutral Axis Depth	159
6.4.2	Evaluation of Stress Resultants at the Yield Sections	160
6.4.3	Load Estimate at any Deflection Stage	162
6.5	Numerical Study	163
6.5.1	Comparison between Enhanced carrying Capacities Obtained by Various Design Methods	163
6.5.2	The Influence of Coefficient of Orthotropy ( $\mu$ ) on the Enhanced Load Carrying Capacity of the Slab	165
6.6	Conclusions	168
CHAPTER VII	EXPERIMENTAL INVESTIGATION	
	NOMENCLATURE	182
7.1	INTRODUCTORY REMARK	183
7.2	Simulation of Edge Condition for the Slabs	185
7.3	Details of the Slab Panels	186
7.4	Casting of Models	187
7.5	Testing	188
7.6	Discussion of Results	190
7.7	Conclusions	192

CHAPTER VIII	FLAT SLABS WITH MINIMUM VOLUME OF REINFORCEMENT	
	NOMENCLATURE	215
8.1	INTRODUCTORY REMARK	218
8.2	Flat Slab on Point Supports	218
8.2.1	Displacement Field and Yield Moment Capacity of the Slab	220
8.2.2	Load-Transmission	221
8.2.2.1	First system of load transmission	222
8.2.2.2	Second system of load transmission	223
8.2.3	Moment capacity Volume of the Slab (Using the assumed moment field)	225
8.2.4	Moment Capacity Volume of the slab (Using the displacement field)	227
8.2.5	Minimum Moment Capacity Volume	<b>230</b>
8.3	Flat Slab on Finite Column Supports	231
8.4	Discussion of Results	235
8.4.1	Flat Slab on Point Supports	235
8.4.2	Flat Slab on Finite Column Supports	238
8.5	Conclusions	241
CHAPTER IX	CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK	
9.1	CONCLUSIONS	298
9.2	Recommendations For Future work	300
	BIBLIOGRAPHY	302