

ON THE OPTIMISATION OF ROTATING FLOAT IN THE
CLOSED LOOP STOCHASTIC INVENTORY SYSTEMS

A Thesis submitted in partial fulfilment of the
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in
Mechanical Engineering

by
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CERTIFICATE

This is to certify that the Thesis entitled ' On the optimisation of rotating float in the closed loop stochastic inventory systems' being submitted by Mr. Prem Vrat to the Indian Institute of Technology, New Delhi for the award of the degree of the Doctor of Philosophy in Mechanical Engineering is a record of the bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for the submission of this Thesis, which has reached the requisite standard.

The results contained in this Thesis have not been submitted in part or in full, to any other University or Institute for the award of any other degree or diploma.

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ABSTRACT

This Thesis deals with the development and analysis of a mathematical model for the optimisation of rotating float in a closed loop stochastic inventory system. The characteristics of such a system are discussed. Previous work on this topic is reviewed and classified according to queueing, reliability, simulation and inventory theoretic approaches.

The mathematical model developed for Erlang inter-arrival time and ~~and~~ Erlang overhaul cycle time distributions enables the study of the operating characteristics of the system for various input-output situations (varying from deterministic to stochastic). The behaviour of the system as obtained by the model has been employed for developing a cost-model in order to minimise the total system cost consisting of the cost of carrying inventory and the cost of shortage. The utility of the model has been illustrated by an example for the optimisation of the spare float of air-craft engines in a maintenance base. The sensitivity of the system to changes in various parameters has been examined. The effect of variability of the overhaul cycle time on the optimal float and on the total system cost has been determined. The concept of system entropy has been introduced in order to analyse the effect of uncertainty on the optimal float and to establish a relationship between the degree of control and the system disorder. The model has been further utilised for determining the effect of increased shop output rate

on the optimal float and on the total system cost.

The model developed has also been verified by simulating the system under applicable conditions. A relatively simple but useful approach termed by the author as 'TTDD approach', has also been suggested for the float determination.

The inventory model developed has been further applied to a number of non-inventory type queueing systems and the conceptual similarity between these systems and the system under study has been brought out. It is felt that the model developed can be an alternative to the queueing theory for the analysis of $E_L/E_K/C$ queueing systems. In particular, the problems of machine assignment and interference have been solved using the model developed under centralised as well as decentralised service policies. The effect of pooling up of the facilities has been analysed.

Finally, the suggestion has been made to extend the model for situations exhibiting the erratic pattern of arrivals and Erlangian overhaul cycle times. An approximate model has been developed for erratic demand pattern. Guidelines for further investigations have been outlined.

A major part of the work reported in this thesis has already been published by the author.

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