

THE GENERALISED ROTATING FIELD THEORY
OF ELECTRICAL MACHINES

by

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C E R T I F I C A T E

Certified that the dissertation entitled " The Generalised Rotating Field Theory of Electrical Machines" which is being submitted by Shree S.Sreenivasa Murthy in partial fulfilment for the award of the Degree of Doctor of Philosophy in Electrical Engineering of the Indian Institute of Technology, Delhi is a record of the student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.


(G.S. Vha)

Dated 1974.

TO

MY PARENTS

who always encouraged me to know more

ACKNOWLEDGMENT

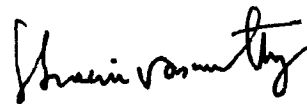
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SUMMARY

The generalised approach to the study of the electrical machine theory relies heavily on the classical two-reaction theory first developed for the synchronous machine by Blondel, Doherty, Parks and others. This approach has by now been so well established that the possibility of using the rotating-field theory as an alternative has attracted little attention. The latter has found favour in the analysis of only induction machines and any significant contribution of the theory to machines other than of the induction type is strictly limited to the classical work of Ku¹⁹ who used it for a generalised analysis of both induction and synchronous machines. Ku's pioneering work did not, however, make much impact because it neither emphasised the relative conceptual simplicity of the rotating field theory as compared to the two-reaction theory, nor did it extend the theory to unbalanced and asymmetrical machines. The basic objective of the investigation reported in this thesis is to develop from simple postulates a rotating-field model for the unified analysis of induction, synchronous and commutator machines.

The main postulate of the rotating field theory is that a field distribution fixed in space and having an amplitude varying sinusoidally with time could be considered to be equivalent to the resultant of two field distributions of constant amplitude but rotating in space in opposite

directions. Generalising this postulate for all the harmonics of the air-gap field, Brown & Jha²⁰ developed a generalised theory for induction machines and showed that the well-established symmetrical component theory was but a special case of their generalised theory. They further established that while the symmetrical component theory could be applied only when the winding axes on the machine were successively displaced by $2\pi / m$ electrical degrees where m was any integer, their generalised rotating-field theory suffered from no such restriction. However, since they were concerned primarily with the effect of space harmonics, their generalised model of the induction machine incorporated a symmetrically distributed polyphase secondary winding. This constraint on the nature of the secondary winding severely restricts the use of Brown & Jha's generalised theory and makes it, in its present form, unsuitable for forming the basis of a unified theory of electrical machines.

The first part of the investigation deals with the attempt to modify Brown & Jha's approach to permit the development of a generalised induction machine model with asymmetry on both sides of the air-gap. The interaction of a single-phase winding on the stator excited with a current of frequency f Hz with a single-phase short-circuited winding on the rotor rotating at an angular velocity $2\pi f_r$ rad/sec. is first examined in detail and the volt-ampere equations are established with the help of the generalised rotating-field

parameters. It is shown that the presence of asymmetry in the stator and the rotor leads to the induction of currents of specific frequencies f_1 in the stator and f_2 in the rotor, both f_1 and f_2 are multivalued $f_1 = f \pm k f_r$ where k is even, and $f_2 = f \pm k f_r$ where k is odd. An equivalent circuit is established to represent the behaviour of this 1-1 winding machine. As could be expected, the equivalent circuit contains an unending chain of forward and backward field-loops showing an infinite number of stages of interaction between the stator and the rotor.

The analysis is next extended to a generalised asymmetrical m/n winding machine, i.e. m asymmetrical windings on the stator and n asymmetrical windings on the rotor. No constraints is put on the type of asymmetry present, both the differing number of turns in the phase windings and their non-uniform spatial displacement is included. Voltage equations for stator currents of frequency $f_1 (= f \pm k f_r; k \text{ even})$ and for rotor currents of frequency $f_2 (= f \pm k f_r; k \text{ odd})$ are established in the matrix form. These generalised equations use rotating-field parameters and cover all impressed and generated frequencies, and are applicable to any induction machine configuration operating under any supply condition. Since most practical induction machines have symmetry atleast on one side of the air-gap, the effect of symmetry on the generalised equations is studied in detail. It is shown that the number of generated frequencies reduce considerably when symmetry is introduced on either side of the gap which in turn

reduces the number of independent volt-ampere equations needed to completely specify the behaviour of the machine. The compatibility of this generalised theory with earlier published work dealing with specific types of unbalanced operation is illustrated by showing that the latter are but special cases of the former.

The next part of the investigation deals with the experimental verification of some of those deductions of machine behaviour from the rotating field theory which are not immediately obvious from the two-reaction theory. One of these deductions deals with the presence of several induced frequencies in the stator and rotor windings of asymmetrical machines. Search coil voltages when analysed on a harmonic analyser confirm both the presence of these frequencies and their disappearance on the introduction of symmetry as predicted by the theory. The other important deduction is the possibility of using a specific frequency injection in the stator/rotor winding of an asymmetrical machine to produce synchronising torque at any desired speed. To illustrate this point, since an unsymmetrical rotor of an otherwise symmetrical polyphase induction machine causes induction of frequency $f-2f_r$ in the stator windings, an injection of an additional frequency $f_1=f-2f_r$ would allow synchronous operation at a speed corresponding to f_r . Russel & Norsworthy²⁶ had used this technique in developing a half-speed synchronous motor by injecting direct current in the stator windings

($f_1=0$ hence $f_r=f/2$). Tests in the laboratory confirm that the phenomena is very general and could be used for a speed regulation system for wound rotor induction machines. The quantitative accuracy of the developed rotating field theory is demonstrated through extensive tests on an asymmetrical rotor polyphase induction machine.

The concluding part of the investigation extends the application of the generalised theory developed for the m/n winding induction machine to the synchronous and the commutator machines. A simplified primitive machine model is chosen having a 2-phase asymmetrical winding on the stator and a polyphase symmetrical winding on the rotor. Steady-state voltage equations for the primitive machine model are obtained immediately by substitution from the m/n winding machine equations. A new concept of 'Forward' and 'Backward' currents is introduced while rewriting these equations in operational form. These equations are then shown to be adequate in calculating the behaviour of the synchronous machine under steady-state, transient, and asynchronous operation. Equivalent circuits for the synchronous machine having a conventional field winding or a divided field winding are established both for the synchronous and the asynchronous operation. The results are comparable with those published for the c.w.r and d.w.r. machine earlier. Since the primitive machine model contains two asymmetrical windings on the stator, the theory is directly applicable to any type of asymmetrical

orientation of the field windings.

While extending the analysis to the commutator machines, the similarity and differences in the behaviour of a polyphase wound armature and an armature with a commutator winding are discussed. It is shown that with slight modification and re-interpretation the equations for the primitive machine model are directly applicable to polyphase commutator and d.c. machines. The operational equations of the cross-field as well as the simple direct-current machine are established. Simple substitutions and proper interpretation of terminal constraints lead to the equations for the single-phase series and the repulsion motors.

Since many synchronous and nearly all direct-current machines utilise a salient-pole field construction, the proper accounting of saliency in the rotating-field theory becomes imperative if the theory is to be used as an alternative to the two-reaction theory. It is shown that saliency makes the resultant flux density wave lag behind the resultant mmf wave leading to the development of the well-known reluctance torque. Saliency also produces 3rd harmonic voltages in the windings and develops torque pulsations at frequencies which are multiples of the fundamental frequency.