

IMAGE FORMATION UNDER PARTIALLY COHERENT AND PARTIALLY
POLARISED ILLUMINATION IN APODISED OPTICAL SYSTEMS

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PREFACE

The problem investigated in the present thesis is that of image formation under partially coherent and partially polarised light illumination in apodised optical systems. The imaging characteristics of an optical systems can be modified by controlling the amplitude distribution over the pupil of the system. Such a modification in the amplitude distribution is called apodisation. Generally the purpose of apodisation is to achieve better performance of optical systems for desired applications.

In optical systems like microscopes, the object is illuminated by an auxiliary system consisting of an extended primary source and a system of condenser lenses. The illumination of the object under these conditions is partially coherent. The image forming properties of the imaging system depend upon the impulse response of the imaging system (point spread function), object transmittance function and the coherence conditions prevailing upon the object plane. A mathematical frame-work for treating the problem of image formation has been given by Hopkins[†]. In Hopkins' theory, the illumination at the object plane is considered as due to an incoherent effective source such that the source intensity distribution function and the mutual coherence function at

[†] H.H. Hopkins, Proc. Roy. Soc. A 208, 263 (1951).

the object plane form a Fourier transform pair in accordance to the Van Citteret Zernike theorem. Under conditions which are usually met in practice, the exit pupil of the condenser is illuminated incoherently and may be known as the effective source. The state of coherence in the object plane is given by a parameter ' ρ ' defined as the ratio of numerical aperture of the condenser to that of the objective. The coherent and incoherent illumination limits correspond to $\rho \rightarrow 0$ and $\rho \rightarrow \infty$ respectively.

In case of image formation through a telescopic system, point spread function (PSF) is modified from its ideal shape due to atmospheric turbulence in the atmosphere. As Hopkins' theory of image formation was not applicable to deal with the intensity distribution in the PSF/far field diffraction patterns, the theory of coherence was developed further which led to Schell's theorem. Schell's theorem⁺⁺ facilitates the calculation of intensity distribution in the far field diffraction patterns. The intensity distribution in the far field diffraction pattern depends on the incoherent optical transfer function of the imaging system and partially coherent field at the entrance pupil. So in order to evaluate the intensity distribution in the diffraction pattern, OTF of the imaging system has to be worked out. Determination of OTF has also become very important means of evaluating the

⁺⁺ A.C. Schell, Ph.D. Theses, Massachusetts Institute of Tech(1961).

performance of incoherently illuminated optical systems. In the calculation of diffraction patterns, normalised mutual intensity of illumination which account for the loss of coherence can be considered as a second cascading system to incoherently illuminated telescopic imaging system.

The diffraction spread of the optical system masked with polarisation masks in different zones of the system depends on the polarisation state of the incident radiation. By making suitable choice of the state of polarisation and with proper orientation of the polarisation masks, one can simulate the conditions of different amplitude and phase coatings in different zones of the optical system. The analyser in the image plane further modifies the diffraction pattern and hence the optical transfer function of the optical system. Therefore, this is very effective technique of achieving apodisation. Study of partial polarisation has a great deal in common with partial coherence since both are related to the statistical nature of incident radiation. Several matrix methods are available to study the image formation under partially polarised light.^{†,††} In coherency matrix approach, coherency matrix describes the state of polarisation. This matrix is modified by 2x2 Jones matrix

+ R.C. Jones, J.Opt.Soc.Am. 46,126 (1956)

++ G.Parrent and P Roman, Nuovo.Cimento 15 , 370 (1960)

representing the optical system.

First we consider image formation of extended objects in apodised microscope. The non-periodic extended objects like straight edge, slit and bar are very much used for evaluation of optical systems. Hopkins' theory has been applied by a number of authors in studying the images of extended objects for uniformly illuminated aperture. However, a little work has been done on the images of extended objects employing apodised apertures. In view of the importance of these type of studies, we have investigated the images of straight edge, slit and bar objects employing apodised apertures. The objects considered are having amplitude as well as phase variations. Two different types of apodisation filters have been considered. In the first type, the amplitude is maximum at the centre and tapers off towards its edges, the reverse is true for the second type. The image formation by an aberration free annular aperture has also been considered.

The influence of condenser transmission characteristics upon the performance of microscopic systems has also been investigated by following the same approach. By having amplitude coatings over the condenser aperture, the state of coherence at the object plane and hence the image forming properties of the system can be modified. Three different

effective source intensity distributions which fall into two types have been considered. The first type of intensity distribution gives rise to improvement in image contrast at the cost of little decrease in image resolution and the second type gives improvement in image resolution at the cost of little decrease in image contrast.

The use of centrally obscured condenser is very common in microscopic systems. The effect of annular illumination on the images of extended objects has been investigated. It is shown that the annular illumination gives an improvement in resolution in the images. The effect of the combination of annular illumination and second type of intensity distribution has also been investigated. For this combination a significant improvement in image resolution at the cost of little decrease in image contrast is shown which is not possible to achieve by either one alone or by **uniform** illumination.

Next we consider image formation in telescopic system. The atmospheric turbulences limit the performances of telescopic systems. Study of the problem of modifying the far field diffraction patterns by apodised apertures at the pupil of the telescopic systems for improved performances is of much importance. The complex amplitude filters are of much practical use for improving the image quality..

Considering the importance of these filters we have evaluated the optical transfer functions of optical systems employing these types of apertures in different zones of the optical system. Very interesting features have been obtained. It has been shown that improvement in the frequency response can be made at low frequencies and/or medium frequencies and/or high frequencies by selecting special cases of complex amplitude filters in different zones. Further, some of the results of OTFs obtained are used in calculating the far field diffraction patterns by telescopic systems. To account for the effect of atmospheric turbulence, Gaussian form of co-rrrelation has been applied. The results of the far field diffraction patterns and two point resolution studies have been compared with that of uniformly illuminated aperture.

We have also studied the point image intensity distributions and optical transfer function of an optical system masked with two and three linear polarisation masks. The incident illumination has been considered fully elliptically polarised beam. The effect of the orientation of the analyser and polarisation form of the incident illumination has been taken into account. We have also considered the case of partially polarised incident illumination by applying coherency matrix formulation. The point image intensity distributions

of an optical system masked with two partial circular polarisers has been worked out by making use of coherency matrix approach. Three different configurations of partial circular polarisers considered are (I) Right circular polariser in both the zones, (II) Left circular polariser in both the zones and (III) Right circular polariser in one zone and left circular polariser in the other zone. The state of polarisation in the image plane for above configurations has also been worked out. We have also considered the optical system masked with two polarisers P_1 and P_2 of partial linear and/or partial circular type in the two zones. Three different configurations have been considered - (I) Partial linear polariser in both the zones, (II) Partial circular polariser in the inner zone and partial linear polariser in the outer zone and (III) Partial linear polariser in the inner zone and partial circular polariser in the outer zone.

The work presented in the thesis has resulted in the form of following publications :-

1. Frequency response of apodised systems, *Optica Acta* 21, 737 (1974).
2. Frequency response of complex amplitude annuli, *Ind. J. Pure and Appl. Phys.* 13, 116 (1975).
3. Effect of atmospheric turbulence on the performance of optical systems with generalised annular apertures, *Jap. J. Appl. Phys.* 14, 983 (1975).

4. Images of semitransparent phase edges by coherently illuminated optical systems, J. Ind. Nat Sci. Acad. 39, 314 (1973).
5. Diffraction images of coherently illuminated objects in the presence of spatial frequency amplitude filters, Ind. J. Pure and Appl. Phys. 12, 133 (1974).
6. Influence of an apodised condenser in a microscope upon images of slit and bar objects, Optik 41, 25 (1974).
7. Images of semitransparent phase objects by coherently illuminated optical systems, J. Opt. 4, 37 (1975).
8. Influence of an obscured condenser in a microscope upon images of extended objects, J. Opt. 4, 32 (1975).
9. Influence of an apodised condenser in a microscope upon images of extended objects, Ind. J. Pure and Appl. Phys 14, 372 (1976).
10. Imagery of extended objects by an annular aperture under partially coherent illumination, Jap. J. Appl. Phys. 15, 1709 (1976).
11. Influence of an apodised condenser in a microscope on images of extended objects (Annular Source), Ind J. Pure and Appl. Phys. 15, 188 (1977).
12. On the performances of apodised optical systems, J. Opt. 7, 90 (1978).

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