

**DYNAMIC ANALYSIS OF BIMODULAR MATERIAL
LAMINATED PLATES AND SHELLS**

by

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Department of Applied Mechanics

*Submitted
in fulfillment of the requirements of the degree of*

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**DEPARTMENT OF APPLIED MECHANICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
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Dedicated
to
my father
and
friend late Shishu Ranjan Kundu

CERTIFICATE

This is to certify that the thesis entitled “**Dynamic Analysis of Bimodular Material Laminated Plates and Shells**” being submitted by Mr. Kallol Khan to the **Indian Institute of Technology, Delhi** for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a record of original, bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

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(Kallol Khan)

ABSTRACT

The different stress-strain relation in tension and compression, exhibited by many of the composites, can be represented by bimodular constitutive model. Laminated rectangular plates and cylindrical/conical panels/shells made up of such composites are widely used in engineering applications. These structural elements are often subjected to dynamic loading, and therefore, it is important to study their dynamic characteristics viz. free vibration frequencies, mode shapes and forced response. The analysis of bimodular laminated structures is more difficult to accomplish than that of unimodular material laminated structures since elastic moduli depend on stresses/strains which are unknown *a priori*. The investigation of dynamic characteristics of bimodular material laminated shells and panels has not received the adequate attention of researchers. Further, the steady state frequency response of bimodular structures is not dealt in the available literature. An attempt has been made in the thesis to investigate the free- and forced vibrations of bimodular material laminated composite cylindrical/conical panels and cylindrical shells.

A new constitutive model for bimodular material laminated structures is proposed based on fiber direction strain based Bert model and weighted compliance model of Jones to account for fiber as well as transverse to fiber direction strains while assigning the material properties. The study is carried out using field consistent finite element and first order shear deformation theory. An iterative eigenvalue approach is used for obtaining the free vibration frequencies for positive and negative half cycles for each mode. The Galerkin time domain approach is developed for steady state frequency response analysis of bimodular structures under harmonic excitation. The analytical solutions for free and forced vibrations are also obtained using layerwise higher order theory and the results are compared with those of the

first order shear deformation theory for simply supported bimodular material cross-ply laminated plates and cylindrical panels. It is worthwhile to mention that the dynamic analysis of bimodular structures is computationally quite involved and challenging.

The detailed parametric study is carried out to investigate the effects of thickness ratio, ply-angle, aspect ratio, semi-cone angle, bimodularity ratio and boundary conditions on vibration characteristics of cylindrical/conical panels and cylindrical shells.

The main contributions of the thesis are development of a new constitutive model, solution methodology based on Galerkin time domain approach for steady state forced response of bimodular structures subjected harmonic excitation, free and forced vibration analyses of bimodulus cross- and angle-ply laminated cylindrical- and conical panels/shells, study on the effect of bimodularity ratio on vibration characteristics of cylindrical and conical panels. Several new results are presented in the thesis. Unlike unimodular materials, the vibration frequencies and distribution of deflection/stresses/strains are found to be significantly different in positive and negative half cycles of motion of bimodular material laminated plates and shells. It is expected that the results presented in the thesis may be of interest to the scientists and engineers working in the area of contemporary structural mechanics.

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