

**CONTINUUM DAMAGE MODELING OF COMPOSITE  
LAMINATED PANELS**

by

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*in fulfillment of the requirements of the degree of*

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# Certificate

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This is to certify that the thesis entitled “**Continuum Damage Modeling of Composite Laminated Panels**” being submitted by **Mr. Ankur Kumar Gupta** to the **Indian Institute of Technology Delhi** for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a record of original, bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

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(Ankur Kumar Gupta)

# Abstract

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Continuum damage mechanics is a phenomenological approach that represents the macroscopic effects of microscopic cracks/cavities on the material stiffness and strength by introducing internal state variables referred to as damage variables ranging from zero implying an undamaged material to unity implying a completely damaged material. It is concluded from the literature review that the combined effect of evolving continuum damage and geometric nonlinearity on the progressive damage, load carrying capacity and stress distribution has not been investigated for laminated composite plates/panels subjected to transverse/in-plane loading. Further, the application of global higher order theories and/or nonlocal effects for progressive damage analysis of thick composite laminated structures is not explored. The main contributions of the thesis are: finite element formulation incorporating thermodynamically consistent continuum damage model and geometric nonlinearity, progressive damage analysis of composite laminated plates and cylindrical/conical panels under transverse and in-plane loadings, and testing the efficacy of global higher-order shear deformation theories for thick composite laminated plates/panels with local/nonlocal continuum damage models.

The study is carried out using first and higher-order shear deformation theories based continuum damage mechanics model within the framework of irreversible thermodynamics with/without inclusion of geometric nonlinearity. For the discretization of displacement field variables, field consistent eight noded isoparametric finite element for 2D theories and 27 noded three dimensional isoparametric finite element for 3D analysis are used. The nonlinear governing

equations are solved using Newton–Raphson iterative technique coupled with the adaptive displacement control method to trace the equilibrium path. The damage evolution equations are solved at every Gauss point using Newton-Raphson iterative technique within the iterations of each loading/displacement increment.

Several new results are presented for failure load, damage/stress distribution and static/postbuckling response of cross-/angle-ply laminated plates and cylindrical/conical panels with different boundary conditions and geometrical parameters. It is found that the failure load of immovable simply supported plates under transverse and cylindrical panels under inward radial distributed loadings depict decreasing-increasing-decreasing trend with the increase in the span-to-thickness ratio. The combined influence of bending and middle surface stretching on evolving damage significantly changes the stress distribution, and the magnitudes of transverse to fiber direction normal/in-plane shear stresses decrease significantly compared to those without damage. The ratio of failure load to buckling load increases with the increase in thickness ratio due to the dominating restoring action of inplane stretching forces in the postbuckling region of thin plates. The failure load of antisymmetric cross-ply cylindrical panels with the fibers along circumferential and meridional directions in inner and outermost layers, respectively, is significantly greater compared to that for panels with reverse lamination scheme. The nonlocal damage models reveal better convergence characteristics with the nonlocal scale parameter in the range of 0.5 – 3% of plate/panel dimensions.

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