CHARACTERIZATION OF CAPILLARY DRIVEN FLOW IN LAYERED POROUS MEDIA

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CHARACTERIZATION OF CAPILLARY DRIVEN FLOW IN LAYERED POROUS MEDIA

by

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To my parents and mentors

for their unwavering guidance and love.

Knowledge makes one humble, humility leads to worthiness,

worthiness creates wealth and enrichment,

enrichment leads to right conduct,

right conduct brings joy and contentment.

— *Hitopadeśa*

Certificate

This is to certify that the thesis entitled **"Characterization of capillary driven flow in layered porous media"**, submitted by **Akshit Agarwal** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of original research work carried out by him under my supervision in conformity with rules and regulations of the institute. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Date: July 24, 2025

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- Akshit Agarwal

Abstract

Flow in porous medium refers to the movement of fluids through materials with interconnected pores, such as hydrocarbon reservoirs. Capillary driven flow, also called as spontaneous imbibition (SI) occurs when a fluid-filled solid is brought in contact with another fluid which preferentially wets the solid, therefore the wetting fluid is drawn into a porous medium and displaces the resident non-wetting fluid by capillary action. It is useful in many applications of soil science, geological carbon sequestration, 2-Dimensional (2D) or 3-Dimensional (3D) paper microfluidics, and recovery of oil from naturally and artificially fractured reservoirs.

In a single capillary, Lucas (1918) and Washburn (1921) observed that the capillary driven flow phenomenon is characterized by diffusive dynamics. However, for a heterogeneous porous medium consisting of two or more layers, the capillary-controlled displacement cannot be simply explained using Washburn's law. In this work, the effect of different parameters on the displacement of oil by water in layered porous media due to capillarity was investigated. For two-layered media, the influence of varying layer properties on cross-flow between the layers and the resulting changes in flow behavior was demonstrated. Specifically, a Darcy-scale model incorporating porosity (ϕ), permeability (k), relative permeability (k_r), and capillary pressure (P_c) – saturation (S_w) relationships was employed as the governing flow parameters within the layers of the porous medium. One-dimensional (1-D) simulations were performed using the MATLAB Reservoir Simulation Tool (MRST), an open-source simulator, to study co-current spontaneous imbibition in both homogeneous

and layered porous media. The study focused on the movement of wetting fluid saturation and the evolution of the front location during the displacement process, wherein a significant amount of residual non-wetting fluid remained trapped within the layers.

The numerical model for homogeneous porous media was validated under three different wettability conditions by comparing the obtained saturation profiles with a semi-analytical solution available in the literature. A close agreement between the numerical results and the semi-analytical solution was observed only in the case of mixed-wet porous media. These findings highlight the necessity of numerical simulations at the Darcy scale, as the available analytical models for spontaneous imbibition in homogeneous porous media may not yield reliable results for arbitrary relative permeability and capillary pressure–saturation functions.

The results demonstrate that cross-flow between the layers is not confined to the region near the imbibition front as was believed earlier. Comparing imbibition in the interacting and non-interacting porous layers, we show that (a) porosity or permeability variation results in cross-flow, causing the saturation profiles of the layers to approach the saturation profile observed in an averaged porosity or averaged permeability porous medium, (b) capillary pressure variation results in cross-flow in different directions at different locations to equalize P_c between layers, (c) when both capillary pressure and permeability are correlated, an anomalous behavior is observed where the low permeability layer exhibits the leading front, (d) when both capillary pressure and porosity are correlated, the high porosity layer exhibits the leading front, (e) when porosity and permeability are correlated, the flow is dominated due to the permeability of the layers; the interaction in layers causes the fronts to come closer, (f) when porosity, permeability and capillary pressure are correlated, a contrasting behavior is observed where the fluid front leads in the low permeability layer except near the front, and (g) when different wetting porous layers interact, the cross-flow causes the front in both layers to coincide at same location.

Overall, variations in either porosity or permeability—when considered independently—tend to averaging of the flow behavior. However, porosity and permeability are often interrelated and exhibit a strong correlation with the capillary pressure–saturation relationship. When these interdependencies are accounted for, a distinct flow behavior emerges that cannot be accurately captured using averaged properties alone. Understanding this behavior is crucial, as it demonstrates that oil recovery estimates can change considerably when the interactions between layers in a heterogeneous medium are incorporated. Furthermore, the findings of this study provide valuable insights for developing a new model to more accurately predict the overall dynamics of spontaneous imbibition in layered porous media.

सार

परतदार छिद्रयुक्त माध्यम में केशिका संचालित प्रवाह का लक्षण वर्णन

छिद्रयुक्त माध्यम में प्रवाह का तात्पर्य तरल पदार्थों के उन पदार्थों के माध्यम से संचलन से है जिनमें आपस में जुड़े हुए रंध्र (pores) होते हैं, जैसे कि हाइड्रोकार्बन भंडार (reservoirs)। केशिका क्रिया द्वारा प्रवाह, जिसे स्वैच्छिक इम्बाइबिशन (spontaneous imbibition, SI) भी कहा जाता है, तब होता है जब किसी तरल से भरे ठोस को ऐसे तरल के संपर्क में लाया जाता है जो उस ठोस को प्राथमिक रूप से गीला करता है; परिणामस्वरूप, गीला करने वाला तरल केशिका बल द्वारा माध्यम में खिंचता है और वहां पहले से मौजूद गैर-गीले तरल को विस्थापित कर देता है। यह घटना मिट्टी-विज्ञान, कार्बन संग्रहण, 2D या 3D पेपर माइक्रोफ्लुइडिक्स, और प्राकृतिक या कृत्रिम रूप से फ्रैक्चर किए गए भंडारों से तेल निकालने जैसे अनुप्रयोगों में अत्यंत उपयोगी है।

एकल केपिलरी के संदर्भ में, लुकास (1918) और वाशबर्न (1921) ने पाया कि इस प्रकार का प्रवाह विसरणीय (diffusive) गतिकी दर्शाता है। हालांकि, जब माध्यम में दो या अधिक परतें होती हैं, तो वाशबर्न के नियम से प्रवाह का सटीक वर्णन संभव नहीं होता। इस कार्य में, परतदार माध्यम में पानी द्वारा तेल के विस्थापन पर विभिन्न मापदंडों के प्रभाव का अध्ययन किया गया है। दो-परतीय माध्यम के लिए, यह दर्शाया गया है कि परतों के गुणों में भिन्नता परतों के बीच क्रॉस-फ्लो उत्पन्न करती है, जिससे प्रवाह व्यवहार में परिवर्तन होता है। इसके लिए डार्सी स्तर का एक मॉडल उपयोग में लिया गया है, जिसमें छिद्रता (ϕ), पारगम्यता (k), सापेक्ष पारगम्यता (k_r), और केशिका दबाव (P_c) – संतृप्ति (S_w) संबंधों को प्रवाह मापदंडों के रूप में शामिल किया गया है।

1D सिमुलेशन हेतु MATLAB Reservoir Simulation Tool (MRST), एक ओपन-सोर्स सॉफ्टवेयर का उपयोग किया गया, जिससे समरूप (homogeneous) और परतदार (layered) छिद्रयुक्त माध्यम में सह-दिशात्मक स्वैच्छिक इम्बाइबिशन का अध्ययन किया गया। ध्यान विशेष रूप से गीले तरल की संतृप्ति की गति और प्रवाह अग्रिम की स्थिति के विकास पर केंद्रित रहा, जहाँ एक महत्वपूर्ण मात्रा में गैर-गीला तरल विस्थापित हुए बिना परतों में फंसा रहता है।

समरूप माध्यम के लिए संख्यात्मक मॉडल को तीन भिन्न वेटबिलिटी स्थितियों में साहित्य में उपलब्ध अर्द्ध-विश्लेषणात्मक समाधान से तुलना कर सत्यापित किया गया। केवल मिश्रित-गीला (mixed-wet) माध्यम के लिए संख्यात्मक और अर्द्ध-विश्लेषणात्मक परिणामों में निकटता पाई गई। यह दर्शाता है कि

डार्सी स्केल पर SI के लिए मौजूदा विश्लेषणात्मक मॉडल मनमाने सापेक्ष पारगम्यता और $P_c - S_w$ कार्यों के लिए पर्याप्त नहीं हैं।

परिणामों से यह ज्ञात हुआ कि परतों के बीच क्रॉस-फ्लो केवल इम्बाइबिशन फ्रंट के समीप तक सीमित नहीं होता। इंटरैक्टिंग और नॉन-इंटरैक्टिंग परतों की तुलना में पाया गया कि:

(a) छिद्रता या पारगम्यता में अंतर से क्रॉस-फ्लो उत्पन्न होता है, जिससे दोनों परतों के संतृप्ति प्रोफाइल औसत गुणों वाले माध्यम की तरह हो जाते हैं,

(b) P_c में अंतर परतों के बीच विभिन्न दिशाओं में क्रॉस-फ्लो उत्पन्न करता है ताकि संतुलन स्थापित हो,

(c) यदि P_c और पारगम्यता सह-संबंधित हैं, तो कम पारगम्यता वाली परत अग्रिम प्रवाह दर्शाती है,

(d) यदि P_c और छिद्रता सह-संबंधित हैं, तो अधिक छिद्रता वाली परत अग्रणी रहती है,

(e) जब छिद्रता और पारगम्यता सह-संबंधित होती हैं, तो प्रवाह पारगम्यता द्वारा नियंत्रित होता है और परतों का इंटरैक्शन अग्रिम को समीप लाता है,

(f) जब तीनों गुण (छिद्रता, पारगम्यता, और P_c) सह-संबंधित होते हैं, तो प्रवाह अग्रिम कम पारगम्यता वाली परत में अग्रणी होता है सिवाय अग्रिम सीमा के निकट,

(g) विभिन्न वेटबिलिटी वाली परतों के इंटरैक्शन से दोनों परतों में अग्रिम प्रवाह एक स्थान पर संयोग करता है।

कुल मिलाकर, छिद्रता या पारगम्यता में स्वतंत्र रूप से परिवर्तन प्रवाह के औसत व्यवहार की ओर ले जाता है। लेकिन, ये गुण अक्सर एक-दूसरे से जुड़े होते हैं और $P_c - S_w$ संबंध से भी प्रभावित होते हैं। जब इन अंतःसंबंधों को शामिल किया जाता है, तो प्रवाह का एक विशिष्ट व्यवहार सामने आता है जिसे केवल औसत गुणों के आधार पर नहीं समझा जा सकता। यह अध्ययन तेल पुनर्प्राप्ति अनुमानों को बेहतर बनाने और परतदार माध्यम में SI के सटीक भविष्यवाणी के लिए एक नया मॉडल विकसित करने की दिशा में मार्गदर्शन प्रदान करता है।

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Abbreviations

SI	Spontaneous Imbibition
1-D	One Dimensional
2-D	Two Dimensional
3-D	Three Dimensional
COVID-19	Coronavirus disease 2019
CCS	Carbon Capture and Storage
CO_2	Carbon dioxide
VOF	Volume of fluid
SWW	Strongly Water Wet
WWW	Weakly Water Wet
MW	Mixed Wet

Symbols

A	Parameter characterizing the rate of SI, $(m/\sqrt{s})$
D	Capillary dispersion coefficient, m^2/s
k_h	Absolute horizontal permeability (in the x-direction), millidarcy
k_{rj}	Relative permeability of the phase, dimensionless
$k_{rj,max}$	Endpoint relative permeability
P_c	Capillary pressure, Pa
S	Saturation, dimensionless
q	Darcy velocity, m/s
$P_{c,entry}$	Entry capillary pressure, Pa
n_j	Exponential parameter for the phase relative permeability curve
β_{pc}	Exponent of the capillary pressure, P_c for the oil-water system
μ	Fluid viscosity, Pa.s
f_w	Water fractional flow for viscous-dominated (or Buckley-Leverett) flow, dimensionless
F	Capillary controlled fractional flow, dimensionless
$F^{'}$	Derivative of F wrt water saturation
$F^{''}$	Second derivative of F with respect to water saturation

ϕ	Absolute porosity
t	Time, seconds
x	Distance, m
J	J-function for Leverett J correlation to calculate P_c
σ	Interfacial tension, N/m
θ	Contact angle
λ_w	Water mobility $= k_{rw}/\mu_w$
λ_o	Oil mobility $= k_{ro}/\mu_o$
Q	Volumetric flow rate, m^3/s
Q_w	Cumulative volume imbibed / injected, m