NUMERICAL STUDY OF MULTI-SIZE PARTICULATE FLOW IN HORIZONTAL RECTANGULAR DUCTS AND PIPES

by

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CERTIFICATE

This is to certify that the thesis entitled **NUMERICAL STUDY OF MULTI-SIZE PARTICULATE FLOW IN DUCTS AND PIPES** being submitted by **Jupudi S. Ravichandra** to the **Indian Institute of Technology, New Delhi (India)** for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a bonafide research work carried out by him under our supervision and guidance. The research reports and the results presented in this thesis have not been submitted in parts or in full to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

Galerkin finite element method (GFEM) is developed and implemented into a FORTRAN code to study dense slurry flow with broad particle size distribution. The code is developed entirely from scratch as part of this study. Developing flow in two-dimensional horizontal closed channels and fully developed flow in horizontal pipes are studied. Several size classes account for the breadth of the particle size distribution. Each size class (called species or phase) is assigned with a representative particle diameter and a corresponding part of the overall concentration. The particulate phases (species) and the carrier phase are treated as coexisting continua that exchange momentum and energy with each other. In other words, the Eulerian - Eulerian approach is adopted. The continuity and momentum equations are obtained for each 'phase' by volume averaging. The summation of equations for all the phases yields the governing equations for the pseudo-fluid mixture. The modeled stresses in the solid phase are of four types: viscous, inertial, Coulombic, and collisional. In addition, a drag force between each species and the mixture is considered. The concentration distribution of each species is governed by a convection-diffusion equation. The eddy viscosity (or diffusivity) for each phase is obtained by applying a concentration modification to the eddy viscosity of the pure carrier flow field. The modification is such that both particle-particle and particle-carrier interactions are included. The eddy viscosity of the carrier flow field is obtained *via* either a mixing length or $k-\varepsilon$ model. (Results with both models are shown to be practically identical.) To specify velocity boundary conditions along the walls, wall functions are used for the mixture as well as for each particulate species.

Developing flow through two-dimensional channels is considered mainly to (i) benchmark the performance of GFEM for this class of problems, and (ii) to understand the predominant terms in the momentum equations. The two velocity components of each particulate species and the mixture, the common pressure, and the concentration of each species are the field unknowns for this problem, For computational economy, the velocity field of each particulate species is obtained by particle force balance method (PFBM) in addition to GFEM. It is shown that PFBM

method yields overall converged results in about 30-50% of the computation time required by GFEM for practically identical results.

The numerical algorithm for the channel problem consists of the following steps: (1) solve for the carrier phase flow field; (2) solve for the mixture field by GFEM; (3) solve for the velocity field of each of the species by GFEM or PFBM; (4) solve for the concentration field of the species by GFEM starting from a convection-diffusion equation; (5) steps 3 and 4 are repeated till the solid velocity and concentration field of each of the species converge to a tolerance of 10^{-6} in the infinity norm of the correction vector; (6) steps 3-5 are repeated for all the species of particles; (7) steps 4-6 are repeated till overall concentration (summation over the concentration field of all species) field converges to a tolerance of 10^{-6} in the infinity norm of the correction vector. These steps are repeated until overall convergence is obtained in the mixture flow field and overall concentration. U, V, P formulation (with Q_1Q_0 elements) is used to compute the mixture flow field. The Q_1Q_0 element is used for its computational economy. Newton's iteration and frontal solution technique are employed to generate and solve the system of linear equations. Hughes discontinuous upwinding is incorporated to prevent oscillations in the concentration field for coarse meshes. However, with the fine meshes employed in this study, upwinding is found unnecessary.

Results are validated by mesh refinement study and by comparison with experimental results. Numerical results are found to satisfy mass conservation within 1% for each size class. Predicted fully developed concentration and mixture velocity profiles are in agreement with the corresponding experimental results with a maximum difference of 3.5% and 5% respectively. In the fully developed regime, predicted pressure drop is within a maximum difference of 3.5% with respect to experimental observations. Other salient conclusions from the study of duct flow problem are (1) PFBM is as accurate as GFEM and is 2-3 computationally economical; (2) similarly predictions with eddy viscosity model are as accurate as predictions with $k-\varepsilon$ model, and are computationally economical; (3) Dispersive and supported load stresses are negligible for the operating range studied; and (4) upwinding is unnecessary when sufficiently fine meshes are employed.

In the case of pipe flow, fully developed flow regime is considered. Thus, the axial velocity of the mixture and the vertical velocity and concentration of each particulate species are the field unknowns. Based on the conclusions from the channel problem, PFBM is used in the pipe problem. Essentially, the same sequential approach is adopted for the pipe problem as well. However, nine-node biquadratic isoparametric elements are used to better model the curved geometry. Again predicted results are validated by mesh refinement and by comparison with published experiments of other authors. Fully developed concentration and mixture velocity profiles, and pressure drop in uniform and multi-size particualte flows are found to be mesh independent and are within a maximum deviation of 5% with respect to experimenal results.

The GFEM code developed in this study for fully developed multi-size particulate flow in horizontal pipes is used to generate a huge database for pressure drop and wear rate. Several existing correlations and two semi-analytical methods are tested using the generated database. Further, new correlations and two artificial neural network models (ANN) are also developed for pressure drop. The first model (NN1) uses individual diameters and concentrations of each class size as inputs, and the second (NN2) uses the concentration-weighted mean diameter and standard deviation of particle size distribution as inputs. Both models show comparable pressure drop prediction capability, with NN1 being computationally more expensive. In comparison to the ANN models, the accuracy of predictions using correlations (both those existing in the literature and those developed in this study) and semi-analytical methods is found to be significantly inferior. Regression coefficient over finite element generated data with the best correlation and analytical model is found to be approximately 0.7, whereas it is nearly unity (0.999) with neural network models. Similarly, the average error of prediction over collected experimental data is nearly 20% with the best correlation and analytical model, whereas it is close to 10% with neural network models. This difference is attributed to the inability of correlations and semi-analytical methods to capture the complex nonlinear functional relationships that exist between pressure drop and other operating parameters.

The computed local concentration and particle velocity along the pipe wall are correlated with erosion *via* empirical coefficients available in the literature. Predicted wear trends agree well with experimental results. The uneven wear rate along the pipe wall is characterized by maximum wear rate, its location on the pipe wall, average wear rate and wear rate at pipe bottom. These values are obtained for all the runs performed to generate a database for pressure drop. and a neural network model is developed on the data. Neural network model is found to fit all the four wear variables accurately with a regression coefficient of 0.98.

Thus the combination of GFEM code and neural network models developed in this study serve as an effective tool in the process of designing horizontal pipeline for transport of slurries with broad particle size distribution.

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