ANALYTICAL AND NUMERICAL STUDIES OF CERTAIN PROBLEMS OF FLOWS AND WAVES IN HIGH TEMPERATURE GAS DYNAMICS

By

ANWAR ABDULLA BASSAIF

A THESIS SUBMITTED TO THE
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
FOR THE AWARD OF THE DEGREE OF
DOCTOR OF PHILOSOPHY

Department of Mathematics
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
HAUZ KHAS, NEW DELHI-110016

OCTOBER, 1994

DEDICATED TO THE LOVING
MEMORY OF
MY DEPARTED FATHER

Mr. ABDULLA BASSAIF

CERTIFICATE

This is to certify that the thesis entitled *"Analytical and Numerical Studies of Certain Problems of Flows and Waves in High Temperature Gas Dynamics"* which is being submitted by **Mr. Anwar Abdulla Bassaif** for the award of the degree of **Doctor of Philosophy in Mathematics** to the Indian Institute of Technology, Delhi, is a bonafide record of research work carried out by him under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Rama Shankar 12-10-94

(RAMA SHANKAR)
Associate Professor
Department of Mathematics
Indian Institute of Technology, Delhi
Hauz Khas, New Delhi - 110016, INDIA.

ACKNOWLEDGEMENTS

*I take this opportunity to express my sincere and profound sense of gratitude to my Guide, **Dr. Rama Shankar**, Associate Professor, Department of Mathematics, IIT Delhi, for his able guidance, valuable suggestions and constant encouragement throughout my research work. I am greatly indebted to him for initiating me into research work and cultivating in me a taste for the field of Gas Dynamics. Infact, without his keen interest and kind cooperation, this work would not have been possible.*

*It, also, give me a great pleasure in expressing my sincere regards and gratitude to **Prof. K.N. Mehta**, Head, Mathematics Department, IIT Delhi, **Prof. O.P. Bhutani** and other faculty members of Mathematics Department, IIT Delhi, for their support and help throughout the period of my research work.*

I am thankful to the authorities of IIT Delhi, for providing me the research facilities to undertake this research work.

My thanks also due to the authorities of the University of Aden and the Indian Council for Cultural Relations, for sponsoring my candidature for the award of scholarship for carrying out this project.

To my mother I owe more than I can possibly express. My wife and My daughters Sarah and Saly deserve all praise and sincere appreciation for putting up cheerfully with all the inconveniences and hardships during my absence from home. Finally, I sincerely thank all my friends and colleagues,especially Dr. T.V. Singh, Dr. K.N. Rao and Mr. Dheeraj Bhardwaj for their encouragement and help throughout my studies at IIT Delhi.

(ANWAR A. BASSAIF)

ABSTRACT

The work in this thesis entitled "*Analytical and Numerical Studies of Certain Problems of Flows and Waves in High Temperature Gas Dynamics*" consists of six chapters. Chapter I presents an introduction to the thesis. Chapter II investigates the propagation of acceleration waves in vibrational and radiative nonequilibrium magnetogasdynamic flows. Chapter III deals with the geometry of streamlines in vibrational and radiative nonequilibrium magnetogasdynamics. Chapter IV studies the decay of a sawtooth profile in nonequilibrium magnetogasdynamic flow. Chapter V is concerned with the study of the behaviour of detonation wave in magnetogasdynamic flow. Chapter VI is devoted to study numerically some nonlinear systems of partial differential equations. A brief discussion of each chapter is given below:

CHAPTER I: INTRODUCTION

This introductory chapter presents a brief historical review of the previous studies related to the present work and a discussion concerning the set of equations governing the flows under study is also given.

CHAPTER II: ON THE PROPAGATION OF ACCELERATION WAVES IN VIBRATIONAL AND RADIATIVE NONEQUILIBRIUM MAGNETOGASDYNAMIC FLOWS*

In this Chapter, the effect of magnetic field on the propagation of acceleration waves in a vibrational and radiative nonequilibrium gas flows, which is induced by the motion of a piston advancing with finite acceleration into a constant state of rest, has been studied along with the characteristic path by using the characteristics of the governing quasilinear hyperbolic system of equations of

---

* Published in the International Journal of Engineering Science, Vol. 32, No. 9, pp. 1409-1421 (1994).

the characteristics of the governing quasilinear hyperbolic system of equations of motion as the reference coordinate system. A differential equation governing the growth and decay of an acceleration wave is derived and integrated. The discontinuity at the wave front is shown to satisfy a Bernoulli-type equation which occurs frequently in studies of acceleration waves in continuous media. It is shown that a linear solution in the characteristic plane can exhibit nonlinear behaviour in the physical plane. The critical time when the breakdown of the characteristic solution occurs in the neighbourhood of the leading frozen characteristic is obtained; that is, when all the characteristics will pile up at the wave front to form a shock wave. For the formation of shock waves the following flows; that is,

i) vibrational and radiative nonequilibrium gasdynamic flow;
ii) vibrational nonequilibrium magnetogasdynamic flow;
iii) radiative nonequilibrium magnetogasdynamic flow; and
iv) vibrational and radiative nonequilibrium magnetogasdynamic flow,

have been studied in detail. It has been observed that in the absence of magnetic field the effect of the coupling of vibrational and radiative nonequilibrium characters of the gas is to increase the decay rate of expansion waves, while on compressive waves the effect is to increase the motion of the breakdown point along the leading frozen characteristics, and thus to increase the shock formation time, whereas in the presence of the magnetic field, it has been shown that for the slow nonequilibrium processes the effect of the magnetic field is to slow down the motion of the breakdown point along the leading characteristics, and thus to increase the shock formation time, while the effect on expansion waves is to enhance the decay rate. However, for the quick nonequilibrium processes the effect of the magnetic field on compressive waves is to cause an early shock formation while on expansion waves the effect is to decrease the decay rate.

CHAPTER III: ON THE GEOMETRY OF STREAMLINES IN VIBRATIONAL AND RADIATIVE NONEQUILIBRIUM MAGNETOGAS-DYNAMIC FLOWS

In order to gain some insight into the physics of gas flows with any new model, we have the following main recourses:

i) to investigate some of the classical gas flows with this new model,

ii) to study the geometry of fluid flow with this model.

For the model given in the second chapter, we have studied in this Chapter some aspects of the geometry of fluid flow. More specifically, we are concerned here with the geometry of streamlines in vibrational and radiative nonequilibrium magnetogasdynamics. Infact, the various dynamical and kinematical relations conecting the flow quantities with geometrical parameters of streamline trajectories are formulated here. It is shown that the nonequilibrium character of the gas decreases total pressure gradient along the streamlines, but the trajectories of binormals lie on the surface of the constant total pressure and, if the streamlines are straight lines, the trajectories of the principal normals lie on the surface of the constant total pressure. Further, the vorticity components are found in terms of the curvature of the streamlines, their principal normals and binormals. Finally, a class of circular helical flow is discussed.

CHAPTER IV: ON THE DECAY OF A SAWTOOTH PROFILE IN NONEQUILIBRIUM MAGNETOGASDYNAMIC FLOW*

In this Chapter, a progressive wave analysis has been applied to find an asymptotic solution of the set of nonlinear partial differential equations governing the nonequilibrium magnetogasdynamic flow which is induced by the motion of a piston advancing with finite acceleration into a constant state of rest. The

---

* Published in Int. J. Engng. Sci. Vol. 33, No. 4, pp. 555-563 (1995).

analysis assumes an azimuthal magnetic field produced by a constant current flowing along the axis.An evolution equation is derived for the wave amplitude, which leads to the Bernoulli-type equation governing the growth and decay of acceleration waves. It is found that all expansion waves ultimately decay out, whereas the compressive waves will not. A shock wave becomes weak after travelling a long distance and hence there apears a sawtooth profile with a decaying acceleration wave front on the left end and a weak shock wave on the right end. This physical situation has been analytically investigated. A remarkable difference between the planar and cylindrical wave profiles is pointed out. It has been observed that the length of the sawtooth profile increases faster in the case of a planar wave than in the case of a cylindrical wave. The effect of the magnetic field on the sawtooth profile has been obtained. It is concluded that, for slow relaxation process, the effect of the magnetic field is to enhance the decay rate of increase of the length of the sawtooth profile, whereas for the quick relaxation process the effect is to decrease the decay rate of the profile. However, the effect of the magnetic field on the velocity of the profile is to slow down the velocity decay when the relaxation process is slow, while the effect is to accelerate the velocity decay when the relaxation process is quick.

CHAPTER V: ON THE BEHAVIOUR OF DETONATION WAVE IN MAGNETOGASDYNAMIC FLOW (IN A SPARK IGNITION ENGINE) : A MATHEMATICAL STUDY

In this Chapter, the role of transverse magnetic field on the progress of a plane detonation wave in a polytropic gas is investigated under the assumptions: The gas is shocked by an infinite piston moving at a constant velocity, the shock is sufficient to prepare the charge for ignition and combustion initiated with spark itself takes place rapidly throughout a thin and finite width region in a Spark Ignition Engine. The magnetic field is applied in the transverse direction to the axis of motion of the piston. After the mathematical formulation, the characteristic system and generalised Riemann Invariants, the solution in reaction zone and outside the zone are discussed. Finally, the solution is obtained by matched asymptotic expansion. It turns out that the internal energy at piston decreases with the increase of the Alfvén wave speed, resulting in decrease in the final temperature of the charge in the unburnt zone. Further, the specific energy of formation of combustible products increases as the magnetic field increases. Thus both these factors favour the prevention of knocking in a Spark Ignition Engine.

CHAPTER VI: NUMERICAL SOLUTIONS OF SOME NONLINEAR SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

Nonlinear partial differential equations occur very frequently in mathematical modelling of physical problems arising in science and engineering: for example, wave pheneomena are governed by nonlinear partial differential equations subject to certain initial and/ or boundary conditions. For nonlinear partial differential equations, it is very difficult to find the analytical solutions. Therefore, one has to resort to numerical methods for obtaining their solutions. In this chapter, we have developed some numerical algorithms to solve some of the nonlinear systems of partial differential equations. We divided this Chapter into three sections as follows:

SECTION 1: A STUDY OF DAM-BREAK PROBLEM BY USING CUBIC SPLINES AND OPERATOR-SPLITTING TECHNIQUE*

In this Section, a numerical finite difference scheme based on the technique of operator-splitting and cubic spline functions is developed and used to integrate the one-dimensional shallow-water equations together with initial and boundary conditions. The scheme is applied to the dam break problem, and the computed results are compared with the analytical solution as given by Stoker for

---

* Presented in the Conference on Mathematics for Industrial Development, held at the Department of Mathematics, Institute of Basic Science, Agra University, March 21-24, 1994.

a horizontal and frictionless channel. It is found that the resulting scheme is computationally efficient and produce satisfactory results for the one-dimensional test problem of a breaking dam, especially when supper-critical flow is present, and infact, it can be used to predict accurate solution to the dam break problem in general.

SECTION 2: NUMERICAL SOLUTION OF COUPLED BURGERS' EQUATIONS IN INHOMOGENEOUS FORM*

In this Section, the finite difference scheme based on technique of operator-splitting and cubic spline functions, as developed in section 1, is derived for solving two-dimensional Burgers' equations in "inhomogeneous" form. The scheme is of first order accuracy in time and second order accuracy in space direction and is unconditionally stable. The numerical results are obtained with severe/moderate gradients in initial and boundary conditions and the steady state solutions are plotted for different values of parameters. It is concluded that the resulting scheme works very well even in the case of very severe gradient in the solution. Also the general nature of the scheme provides a wider application in the solution of nonlinear problems.

SECTION 3: A NUMERICAL STUDY OF A CONVERGING CYLINDRICAL SHOCK PROBLEM IN RELAXING GAS FLOW**

By modifying Rusanov's difference scheme as developed for quasilinear hyperbolic system of partial differential equations in quasi-conservative form, a converging cylindrical shock problem in vibrationally relaxing gas has been studied in this Section. By comparing our results with available results in literature for inert gases, the effect of vibrational relaxation on such shock wave has been obtained. It has been shown that cylindrical shock wave in a

---

* Published in the Internationl Journal for Numerical Methods in Fluids, vol. 20, 1263-1271 (1995).

** To appear in Mathematical & Computer Modelling: An Intenrational Journal (1995).

vibrationally relaxing gas decreases in strength as it is propagating towards the axis. It has been observed that the effect of vibrational relaxation is to increase the growth rate of shock strength when it is propagting towards the axis. Further, it has been shown that the vibrationally relaxing character of the gas is to accelerate the shock convergence with axis and thus decrease the convergence time. Finally, by developing a numerical scheme using the splitting-up technique and cubic spline functions in the context of a system of equations, and then applying the scheme for the aforementioned flow, we have obtained the similar effects for the growth rate of shock strength and convergence time.

* * * * * * *

CONTENTS

= = = = = = = = =