

**EFFICIENT LOCALIZATION ALGORITHMS
WITH A FOCUS ON ROBUSTNESS,
HYBRIDIZATION AND OPTIMAL SENSOR
PLACEMENT**

KUNTAL PANWAR



CENTRE FOR APPLIED RESEARCH IN ELECTRONICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

July 2024

©Indian Institute of Technology Delhi (IITD), New Delhi, 2024

**Efficient Localization Algorithms with a Focus on
Robustness, Hybridization and Optimal Sensor
Placement**

by

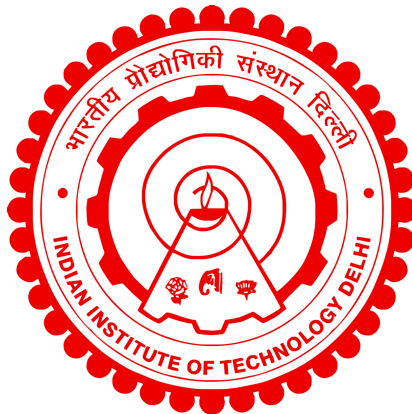
KUNTAL PANWAR

CENTRE FOR APPLIED RESEARCH IN ELECTRONICS

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



**INDIAN INSTITUTE OF TECHNOLOGY
DELHI**

July 2024

Dedicated to my family.

Certificate

This is to certify that the thesis entitled “**Efficient Sensing Methodologies: A Focus on Robustness, Placement, Hybridization, and Other Challenging Aspects**”, submitted by **Kuntal Panwar** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Signal Processing, is a record of the original, bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of our knowledge.

Prof. Prabhu Babu

Thesis Supervisor

Centre for Applied Research in Electronics,
Indian Institute of Technology Delhi.

Acknowledgements

I would like to express my sincere gratitude to all those who have supported and guided me throughout my journey in completing this thesis.

First and foremost, I am deeply thankful to my advisor, *Prof. Prabhu Babu*, for his exceptional mentorship, boundless patience, and constant support. His expertise and dedication have been instrumental in shaping this research.

I also appreciate the contributions of my thesis committee members, *Prof. Arun Kumar*, *Prof. Monika Aggarwal*, and *Prof. Lalan Kumar*, whose insightful feedback significantly enriched this work.

My profound thanks to *Prof. Petre Stoica*, *Prof. Antonia Demaio*, *Prof. Augusto Aubrey*, and *Prof. Paulo Braca* for their guidance and supervision in some of my research papers, which greatly contributed to my academic pursuits.

I am grateful to my colleagues and friends, especially *Astha Saini*, *Ghania Fatima*, *Jasleen Kaur*, *Mayur Katwe*, *Piyush Varshney*, and *Shweta Pal*, for their support and camaraderie, making the research process enjoyable. I would also like to express my heartfelt gratitude to the CARE office staff for their exceptional support and efficiency in handling my paperwork.

I am indebted to my family for their unwavering belief in my abilities and continuous encouragement during my Ph.D. My parents, *Smt. Geeta and Shri Gopal Panwar*, provided unwavering emotional support, and my sister, *Manisha Panwar*, has been my confidante and cheerleader. Their collective support and love were the cornerstone of my resilience, making this achievement possible.

I am also grateful to my aunt and her family for their unwavering support and hospitality during the initial year of my PhD.

This thesis has been a culmination of collective efforts, and I am sincerely grateful to all those who played a part, whether big or small, in its realization.

Thank you all.

Kuntal Panwar

Abstract

The thesis aims to tackle critical challenges within localization techniques, with the overarching goal of enhancing accuracy, efficiency, and robustness across a wide array of applications. The scope of investigation encompasses multistatic target localization, hybrid localization methods, robust localization in the presence of outliers and Non-Line-of-Sight (NLOS) errors, optimal sensor placement strategies, and novel approaches tailored for scenarios lacking transmitter information. The impetus for this exploration arises from the discernible limitations and gaps prevalent in existing methodologies.

In the initial phase, the thesis provides an introductory overview of localization, exploring its wide-ranging applications and discussing a variety of estimation techniques. Additionally, it offers a concise explanation of multistatic localization, underlining the necessity for hybrid localization methods and robust localization schemes. Furthermore, this segment delves into the importance of optimal sensor placement strategies to improve accuracy in localization techniques.

In the subsequent segment, the thesis introduces a foundational framework for optimization algorithms, notably through the exposition of the Majorization Minimization (MM) architecture. Extensions of this architecture, such as Block Majorization Minimization, significantly contribute to the methodological underpinnings of the research.

The third phase delves into the intricacies of multistatic target localization, confronting various challenging scenarios head-on. Initially, an algorithm designed for joint Maximum Likelihood Estimation (MLE) of both the target position and associated noise variances is introduced, especially when prior knowledge of noise variances is absent. This algorithm demonstrates good performance, substantiated by compelling simulation results. Subsequently, an MM algorithm minimizing the Nonlinear Weighted Least Squares (NLWLS) criterion is proposed, showcasing its robustness in handling NLOS conditions and outliers. Additionally, a localization scheme tailored for scenarios lacking transmitter coordinates and synchronization is presented, leveraging Differential Time Delays (DTDs) from each receiver to enhance applicability and effectiveness in high-noise scenarios. Overall, this phase addresses the inherent

challenges of multistatic localization scenarios through innovative methodologies and rigorous validation.

The fourth segment advances the exploration by seamlessly transitioning into hybrid localization methods, which integrate multiple measurements to enhance accuracy and resilience. Specifically, a unified framework for hybrid localization, encompassing Time of Arrival (TOA), Time Difference of Arrival (TDOA), Received Signal Strength (RSS), and Angle of Arrival (AOA) measurements, is introduced to ensure flexibility and accuracy, particularly in scenarios characterized by NLOS errors. Additionally, a robust hybrid scheme amalgamating TOA and RSS measurements is formulated as a NLWLS problem, demonstrating superior accuracy and computational efficiency.

In the fifth part, the thesis delves into optimal sensor placement strategies, which are essential for enhancing source localization accuracy. Centered on addressing the A-optimal criterion design problem within positioning schemes for hybrid localization, the proposed approach not only improves accuracy but also accommodates D and E optimal criteria. Moreover, it addresses complexities associated with both correlated and uncorrelated noise, thus addressing significant gaps in existing methodologies. Additionally, innovative strategies for optimal sensor placement for 3D AOA measurements and Differential Received Signal Strength (DRSS) measurements are also discussed.

In conclusion, the thesis summarizes its substantial contributions and highlights potential avenues for future research. The innovative methodologies and comprehensive exploration presented significantly contribute to advancing the understanding and practical implementation of localization techniques, particularly in navigating the complexities of various challenging scenarios.

सार

यह थीसिस स्थानीयकरण तकनीकों में महत्वपूर्ण चुनौतियों का सामना करती है जिसका लक्ष्य विभिन्न अनुप्रयोगों में सटीकता, दक्षता, और प्रतिस्थापनशीलता को बढ़ावा देना है। इसमें मल्टिस्टैटिक लक्ष्य स्थानीयकरण, हाइब्रिड विधियाँ, बहुतलम में दृढ़ स्थानीयकरण त्रुटियों और गैर-रेखीय दोषों के मौजूदगी में, आदि के लिए अध्ययन किया गया है। इस अन्वेषण का प्रेरणा विद्यमान तकनीकों में दोषों के प्रमुख सीमाओं से होता है।

थीसिस शुरुआती अवलोकन प्रदान करती है, स्थानीयकरण का, जिसमें अनुमान तकनीकों की व्यापक जानकारी और मल्टिस्टैटिक स्थानीयकरण, हाइब्रिड स्थानीयकरण विधियों के लिए आवश्यकता की संकेत की गई है। साथ ही, इस सेगमेंट में सटीकता को बढ़ाने के लिए आदर्श सेंसर स्थान रखने की उपयोगिता के बारे में संक्षिप्त व्याख्या भी है।

थीसिस का दूसरा हिस्सा गहरे रूप से बहुस्थल लक्ष्य स्थानीयकरण की ओर बढ़ता है, जिसमें शोर वेरियेंसेस के पूर्वज्ञान की अभाव में एक लक्ष्य को योग्यता से स्थानीयता प्राप्त करने की चुनौती है। इस संदर्भ में, अनुसंधान ने एक ऐतिहासिक एल्गोरिदम प्रस्तुत किया है जो लक्ष्य की स्थानीयता और संबद्ध शोर वेरियेंसेस की संबंधित ध्वनि विभिन्नताओं की संयुक्त अधिकतम लिखुआ अनुमान (एमएलई) के लिए डिज़ाइन किया गया है, जो प्रेरणादायक सिम्युलेशन परिणामों के साक्षात्कारों के रूप में उदाहरण के रूप में प्रकट होता है। इसके अलावा, मजबूत स्थानीयकरण विधियों की आवश्यकता को स्वीकृति प्रदान करते हुए, अनुसंधान समय ऑफ सिग्नल आगमन (टीएसओए) मापों के लिए रोबस्ट विधियों की ओर बढ़ता है। यहां, एक गैर-रैलियन वेटेड लीस्ट स्क्वेयर्स (एनएलडब्ल्यूएलएस) एल्गोरिदम प्रस्तुत किया जाता है, जो संख्यात्मक सिम्युलेशन में उपस्थित है। ट्रांसमीटर कोऑर्डिनेट्स के बिना मल्टिस्टैटिक लक्ष्य स्थानन को संबोधित करता है, अद्वितीय रूप से डिफरेंशियल टाइम डिलेस (डीटीडी) का उपयोग करके उच्च-शोर स्थितियों में न्यून संवेदकों के साथ बेहतर अनुप्रयोग के लिए। यह आंकड़बाज़ एल्गोरिदम संगतता की गारंटी देता है, एक प्रेसिंग समस्या का समाधान प्रस्तुत करता है और वास्तविक दुनिया की चुनौतियों के प्रति समर्पण को जोर देता है।

अन्वेषण को हाइब्रिड स्थानीयकरण विधियों में सहजता से स्थानीयकरण की ओर बढ़ाता है, कई मापों का संयोजन करके सटीकता और सहिष्णुता को बढ़ावा देने के लिए। विशेष रूप से, अनुसंधान एक मजबूत हाइब्रिड योजना प्रस्तुत करता है जिसमें आगमन के समय (टीओए) और प्राप्त सिग्नल स्ट्रेंथ (आरएसएस) मापों का समाहित स्कीम पेश करता है। एक गैर-रैलियन वेटेड लीस्ट स्क्वेयर्स (एनएलडब्ल्यूएलएस) समस्या के रूप में तैयार की गई इस हाइब्रिड दृष्टिकोण का उत्कृष्ट स्थानीयता और संगणकीय कुशलता दोनों में प्रदर्शन करता है, जो वास्तविक दुनिया के अनुप्रयोगों में एक महत्वपूर्ण पहलू है। इसके अलावा, यह अध्याय हाइब्रिड स्थानीयकरण के लिए समेकित ढांचा पेश करता है, जिसमें टीओए, टाइम डिफरेंस ऑफ आगमन (टीडीओए), आरएसएस, और एंगल ऑफ आगमन (एओए) मापों को शामिल किया जाता है, जिससे एनएलओएस त्रुटियों के साथ भी लचीलापन और सटीकता सुनिश्चित है।

चौथा अंश उच्चतम संवेदक स्थानन प्लेसमेंट रणनीतियों पर ध्यान केंद्रित करता है, ए-ऑप्टिमल मानचित्र डिज़ाइन समस्या के माध्यम से स्रोत स्थानन यथासंभाव सटीकता को बढ़ाता है। इस दृष्टिकोण को डी और ई ऑप्टिमल मानचित्रिता के साथ समर्थन करता है, मौजूदा विधियों में गैप्स को संबोधित करता है। 3D एओए मापण और डिफरेंशियल रिस्वीड सिग्नल स्ट्रेंथ (डीआरएसएस) मापण के लिए रणनीतियां नवाचारी समाधान प्रदान करती हैं, संगणन संघटन और बड़े संवेदक नेटवर्क्स में अनुकूलन की चुनौतियों को ध्यान में रखती हैं। पाँचवा सेगमेंट

समापन में, थीसिस अपने योगदानों का संक्षेप करती है और भविष्य के अनुसंधान के लिए संभावित रास्तों की संकेत सूची देती है। प्रस्तुत की गई नवाचारी विधियाँ चुनौतीपूर्ण परिस्थितियों में स्थानन तकनीकों के प्रैक्टिकल अमल को आगे बढ़ाने में सामर्थ्यपूर्ण योगदान प्रदान करती हैं।

Contents

Certificate	i
Acknowledgements	ii
Abstract	iii
Contents	vi
List of Figures	xiii
List of Tables	xvii
Abbreviations	xix
Notations	xxi
1 Introduction	1
1.1 Various types of measurements	2
1.1.1 Time of Arrival	2
1.1.2 Time Difference of Arrival	4
1.1.3 Received Signal Strength	5
1.1.4 Angle of Arrival	6
1.1.5 Other measurement models	8
1.2 Multistatic Localization	8
1.3 Hybrid Localization	11
1.4 Need for Robust Localization	12
1.5 Optimal Sensor Placement	15
1.6 Motivation	18
1.7 Key Contributions of the Thesis	19
1.8 Thesis Organization	21

2	Mathematical Preliminaries	27
2.1	Majorization Minimization	27
2.1.1	Advantages of MM architecture	29
2.1.2	Convergence of MM	30
2.2	Block Majorization Minimization	31
2.2.1	Convergence of Block MM	32
3	Robust Multistatic Localization	35
3.1	Maximum Likelihood Algorithm for Time-Delay Based Multistatic Target Localization	37
3.1.1	Introduction	37
3.1.2	Data model and Problem Formulation	39
3.1.3	Proposed Method	42
3.1.3.1	Computational complexity and proof of convergence	45
3.1.4	Simulation Results	46
3.1.5	Conclusion	49
3.2	Robust Multistatic Target Localization in the Presence of NLOS Errors and Outliers	50
3.2.1	Introduction and literature review	50
3.2.2	Data model and Problem Formulation	52
3.2.3	Proposed Method	55
3.2.4	Simulation Results	58
3.2.5	Conclusion	62
3.3	Multistatic Target localization in absence of transmitter position using DTD measurements	63
3.3.1	Introduction and literature	63
3.3.2	Data model and Problem Formulation	65
3.3.3	Proposed Algorithm	68
3.3.4	Convergence and Computational Complexity	71
3.3.5	Numerical simulations	72
3.3.6	Conclusion	76
4	Hybrid Localization	77
4.1	Majorization-Minimization based Hybrid Localization Method for High Precision Localization in Wireless Sensor Networks	79
4.1.1	Introduction	79
4.1.2	Related Work	80
4.1.3	Data model and Problem Formulation	84
4.1.4	Proposed Method	87
4.1.4.1	Measurements Weighing	87
4.1.4.2	Proposed algorithm	90
4.1.4.3	Computational complexity and proof of convergence	96

4.1.5	Simulation Results	97
4.1.5.1	CRLB of the hybrid model	99
4.1.5.2	Convergence analysis	101
4.1.5.3	Impact of Measurement Noise	102
4.1.5.4	Impact of Communication Range	103
4.1.5.5	Impact of Anchor Density	105
4.1.5.6	Impact of NLOS paths	106
4.1.5.7	Impact on computational speed	109
4.1.6	Conclusion	109
4.2	A Majorization-Minimization Algorithm for Hybrid TOA-RSS Based Localization in NLOS Environment	110
4.2.1	Introduction and relevant literature	110
4.2.2	Data model and Problem Formulation	111
4.2.3	Proposed Approach	114
4.2.4	Proposed algorithm	114
4.2.5	Computational Complexity, Convergence and Initialization of the proposed algorithm	117
4.3	Numerical Simulations	118
4.3.1	Conclusion	123
5	Optimal Placement Strategies	125
5.1	Optimal Sensor Placement for Hybrid Source Localization Using Fused TOA-RSS-DOA Measurement	126
5.1.1	Introduction and relevant literature	126
5.1.2	Data model and Problem Formulation	131
5.1.3	Proposed Method	138
5.1.3.1	Proposed algorithm for A-optimal design problem	138
5.1.3.2	Proposed algorithm for D- and E-optimal design prob- lem	145
5.1.3.3	Computational complexity and proof of convergence	150
5.1.4	Simulation Results	152
5.1.4.1	Convergence of the proposed algorithm	152
5.1.4.2	Proposed algorithm for the 3D case	154
5.1.4.3	Performance in the presence of uncorrelated noise measurements	156
5.1.4.3.1	Optimal design for uniform sensor-target ranges	157
5.1.4.3.2	Optimal design for nonuniform sensor-target range	160
5.1.4.4	MSE analysis	160
5.1.4.5	Performance analysis of the proposed method for mismatch in the target location estimate	162
5.1.5	Conclusion	163

5.2	Sensor Placement Strategies for Target Localization via 3D AOA Measurements	164
5.2.1	Introduction	164
5.2.2	Data model and Problem Formulation	169
5.2.3	Proposed Method	173
5.2.3.1	Proposed algorithm for A-optimal design problem	173
5.2.3.2	Proposed algorithm for D-optimal design problem	179
5.2.3.3	Computational complexity and proof of convergence	183
5.2.4	Simulation Results	184
5.2.4.1	Convergence Analysis of the Proposed Algorithm	185
5.2.4.2	Sensors Deployment Under Diverse Sensing Scenarios	186
5.2.4.2.1	Uniform target-receiver distance	186
5.2.4.2.2	Non-uniform target-receiver distance	188
5.2.4.3	MSE Analysis	189
5.2.4.4	Robustness Analysis	191
5.2.5	Conclusion	192
5.3	Sensor Placement Strategies for RSSD based Target Localization under constraints	193
5.3.1	Introduction and Literature Review	193
5.3.2	Data model and Problem Formulation	195
5.3.3	Proposed Method	198
5.3.3.1	Computational complexity and convergence	204
5.3.4	Simulation Results	205
5.3.4.1	Convergence Analysis of the Proposed Algorithm	206
5.3.4.2	Deployment of Sensors in Varied Sensing Environments	206
5.3.4.3	MSE analysis	208
5.3.5	Robustness analysis	209
5.3.6	Conclusion	210
6	Conclusion and Scope of Future work	211
6.1	Conclusion	211
6.2	Future scope of work	213
A	CRLB derivation for multistatic target localization problem with unknown noise variance	215
B	Lemma for Chapter 5.1	219
C	Detailed proofs of Lemma 5.7 and 5.8	221
C.0.1	Proof of Lemma 5.7	221
C.0.2	Proof of Lemma 5.8	223

Bibliography	225
List of Publications	263
Technical Biography of Author	265

List of Figures

1.1	TOA measurement model for 3 sensors, where blue \circ represents sensors (s) and orange \circ represents target, and ct is the distance between target and the sensor.	3
1.2	TOA measurement model for 3 sensors, where blue \circ represents sensors (s), green \circ represents common (reference) sensor and orange \circ represents target, and ct is the distance between target and the sensor.	4
1.3	RSS measurement model for 3 sensors, where blue \circ represents sensors (s), and orange \circ represents target, and p shows the power received at the sensor.	6
1.4	RSS measurement model for 2 sensors, where blue \circ represents sensors (s), and orange \circ represents target, θ and ϕ are the azimuth and elevation angles, respectively.	7
1.5	A multistatic system for 2 receivers (denoted by red \circ (s)), 2 transmitters (denoted by blue \circ (T)), and a target (denoted by orange \circ).	9
1.6	Hybrid model for 4 sensors using TOA-TDOA-RSS-AOA measurements in 2D, where blue \circ represents sensors (s), green \circ represents common (reference) sensor and orange \circ represents target. ct is the distance between target and the sensor,	11
1.7	A network consisting of a single target and multiple sensors, with measurements observed at sensors, comprises different types of errors.	13
1.8	A pictorial view of target estimation accuracy enhancement via optimal positioning of sensors.	16
3.1	Multistatic target localization system for $M = 4$ receivers.	39
3.2	Variation of the objective with the number of iterations.	46
3.3	Variation of $\text{MSE}(\mathbf{u})$ with noise power for $K = 30$	48
3.4	Variation of $\text{MSE}(\mathbf{u})$ with K for $\sigma^2 = 20\text{dB}$	48
3.5	MSE variation with no of receivers for $K = 30$ and $\sigma^2 = 20\text{dB}$	48
3.6	An example of multistatic target localization network with $M = 4$ transmitters and $N = 3$ receivers	53
3.7	Simulation results	61
3.8	RMSE variation with no of receivers.	62
3.9	A multistatic target localization system	66

3.10	Variation of objective in (3.51) with iterations.	73
3.11	Variation of MSE with noise variance with and without including [1].	75
3.12	Variation of MSE with number of sensors.	75
4.1	3D WSN: ct_i - distance between source node and i^{th} anchor node, L_i - path loss at i^{th} anchor node and ϕ_i and θ_i - azimuth and elevation angle between source node and i^{th} anchor node $\forall i$	84
4.2	CRLB vs σ for different models	100
4.3	Convergence plots of the proposed algorithms corresponding to different combinations of measurements.	101
4.4	Impact of noise in all measurements on localization accuracy.	102
4.5	Impact of deployment range of anchors on localization accuracy.	104
4.6	Impact of the number of anchors on localization accuracy.	105
4.7	Impact of NLOS noise in measurements.	107
4.8	Impact of the number of NLOS paths in measurements.	107
4.9	Simulation studies of the different methods under different network scenarios	120
4.10	RMSE vs number of anchors	122
5.1	Measurement geometry of the hybrid localization model. The blue triangle represents the target and the black circles are the sensors. p_1, p_2, p_3 and p_4 represent the RSS measurements, $\theta_1, \theta_2, \theta_3$ and θ_4 represent the AOA measurements, and ct_1, ct_2, ct_3 and ct_4 are the measurements corresponding to TOA.	131
5.2	Convergence plot and corresponding sensor placement for 2D hybrid TOA-RSS-AOA under correlated noise. Top: convergence plots; Bottom: sensor placement with black square: target; blue circle: initial sensors' positions; blue pentagram: final sensors' positions.	153
5.3	Convergence plot and corresponding sensor placement for 3D hybrid TOA-RSS under correlated noise. Top: convergence plots; Bottom: sensor placement with black square: target; blue circle: initial sensors' positions; blue pentagram: final sensors' positions.	155
5.4	Optimal sensor trajectories of (a) example 1 and (b) example 2, \circ : sensor initial position, $*$: sensor final position, \square : target position	158
5.5	Optimal sensor trajectories and the final geometries with different sensor initial positions for $m = 2$ and uniform d_i considered in subsection 5.1.4.3.1	158
5.6	Optimal sensor trajectories and the final geometries with different sensor start positions for $m = 3$ and uniform d_i considered in subsection 5.1.4.3.1.	159
5.7	Optimal sensor trajectories and the final geometries for non-uniform d_i in example 3 and 4 considered in subsection 5.1.4.3.2.	160

5.8	A pictorial description of three sensing UAVs localizing an aerial target using 3D AOA measurements where Θ and Φ represent the azimuth and elevation angles, respectively.	169
5.9	Objective function versus iterations.	184
5.10	Optimal placement for 3 receivers uniform noise variance and uniform target-receiver distance.	185
5.11	Optimal placement for 4 receivers non-uniform noise variance and uniform target-receiver distance.	186
5.12	Contour plot of the CRLB trace, assuming 3 receivers at a uniform target-receiver distance and noise variance (network configuration as in Figure 5.10(a)). The point of minimum is shown with red triangle.	187
5.13	Optimal placement for 5 receivers uniform noise variance and non-uniform target-receiver distance.	188
5.14	Optimal placement for 6 receivers non-uniform noise variance and non-uniform target-receiver distance.	188
5.15	A DRSS measurements setup where width of the red lines shows the relative signal strength at that point.	196
5.16	Objective vs iterations for proposed algorithm initialized via uniform placement for 4 sensor in 2D.	206
5.17	Optimal Placement obtained via proposed algorithm for $M = 4$ sensors in 2D	207
5.18	Optimal Placement obtained via proposed algorithm for $M = 6$ sensors in 3D	207

List of Tables

3.1	Pseudo code of the proposed algorithm for problem 3.1	45
3.2	Avg. CPU time for algorithms	49
3.3	Pseudo code of the proposed algorithm for problem 3.2	58
3.4	Computational Complexity and Average Run-Time Comparison	60
3.5	Pseudo-code outlining the proposed algorithm for problem 3.3	71
4.1	Proposed Hybrid Localization algorithm based on the Majorization-Minorization (MM) framework for problem 4.1.	96
4.2	Ranking of hybrid methods for different network scenarios	108
4.3	Comparison of computational time for different combinations of measurements	108
4.4	Pseudo-code of the proposed algorithm for problem 4.2	117
4.5	Average CPU time for Algorithms.	123
5.1	Comparison between related previous studies and proposed Algorithm.	129
5.2	Pseudo-code of the proposed algorithm (A, D and E optimal design) for problem 5.1	149
5.3	Pseudo code to update Φ in A and D optimal design	149
5.4	Pseudo code to update Φ in E-optimal design	150
5.5	Computation complexity of the proposed algorithm for A, D and E optimal designs.	151
5.6	Comparison of theoretical and numerical value of trace of CRLB.	157
5.7	Angles (in degrees) of the optimal sensor-target geometries obtained	159
5.8	Comparison of the MLE performance for different placement.	161
5.9	Comparison of the MLE performance of sensor placement for target location mismatch.	163
5.10	Comparison between some previous studies for optimized sensor placement using AOA measurements and the proposed algorithm.	167
5.11	Pseudo-code outlining the Proposed Algorithm for problem 5.2	183
5.12	Comparison of the MLE performance for different placement strategies.	190
5.13	Comparison of the MLE performance of sensor placement obtained via proposed algorithm in the presence of target location mismatch.	192
5.14	Pseudo-code outlining the Proposed Algorithm for problem 5.3	204
5.15	Comparison of the MLE performance for different placement strategies.	208

5.16 Comparison of the MLE performance of sensor placement obtained via proposed algorithm in the presence of target location mismatch. .	210
---	-----

Abbreviations

AOA	Angle of Arrival
CFWLS	Conventional Fast Weighted Least Squares
CRLB	Cramér-Rao Lower Bound
DRSS	Differential Received Signal Strength
DTD	Delay-Time Difference
FIM	Fisher Information Matrix
GTRS	Generalized Trust Region Subproblem
LOS	Line of Sight
LS	Least Squares
MCC	Matthews Correlation Coefficient
MIMO	Multiple Input Multiple Output
MLE	Maximum Likelihood Estimation
MM	Majorization Minimization
MSE	Mean Squared Error
MUSIC	MUltiple SIgnal Classification
NLOS	Non-Line of Sight
NLWLS	Non-Linear Weighted Least Squares
PCL	Passivet Cohere Localization
RMSE	Root Mean Square Error
RSS	Received Signal Strength
SDP	Semidefinite Programming
SOCP	Second Order Cone Programming
TDOA	Time Difference Of Arrival
TOA	Time Of Arrival
TSOA	Time Sum of Arrival
WLS	Weighted Least Squares

Notations

x	Scalar
\mathbf{x}	Vector
\mathbf{X}	Matrix
x_i	i^{th} element of vector \mathbf{x}
$X_{i,j}$	$(i, j)^{\text{th}}$ element of the matrix \mathbf{X}
$\mathbb{R}^{m \times 1}$	Set of m dimensional vectors of real numbers
$\mathbb{R}^{m \times m}$	Set of $m \times m$ matrices of real numbers
\mathbb{N}	set of natural numbers
\ln	natural logarithm
\log	logarithm with base 10
$\tan^{-1}(x)$	arctangent value of x
$\cos^{-1}(x)$	arccosine value of x
$\sin^{-1}(x)$	arcsine value of x
$(\cdot)^T$	Transpose
$(\cdot)^{-1}$	Inverse
\otimes	Kronecker product
$ \mathbf{X} $	Determinant of matrix \mathbf{X}
$\text{Tr}(\mathbf{X})$	Trace of matrix \mathbf{X}
\mathbf{X}^2	Square of matrix \mathbf{X}
\mathbf{I}_N	Identity matrix of size N
$\text{diag}(\mathbf{x})$	a diagonal matrix with diagonal values from vector \mathbf{x}
∇f	Gradient of a function f
$\nabla^2 f$	Hessian of a function f .
$\frac{\partial(\cdot)}{\partial x}$	first-order derivative with respect to x
$\frac{\partial^2(\cdot)}{\partial x^2}$	second-order derivative with respect to x
$\mathbf{X} \succeq 0$	\mathbf{X} is a PSD matrix

$\mathbf{X} \succeq \mathbf{Y}$	matrix $\mathbf{X} - \mathbf{Y}$ is a symmetric positive semi-definite
$\mathbf{X} \succ 0$	\mathbf{X} is a positive definite matrix
$\lambda(\mathbf{X})$	Maximum eigenvalue of matrix \mathbf{X}
$\ \mathbf{x}\ $	Euclidean norm of the vector \mathbf{x}
$\ \mathbf{x}\ _1$	ℓ_1 norm of the vector \mathbf{x}
$\ \mathbf{X}\ _F$	Frobenious norm of matrix \mathbf{X}
$\mathbf{E}(\cdot)$	Statistical expectation
\circ	Element wise multiplication.
$\text{sgn}(\cdot)$	signum (or sign) function
$p(\mathbf{x})$	probability density function of the random variable \mathbf{x}
$x \sim \mathcal{N}(\mu, \sigma^2)$	x is Gaussian random variable with a mean of μ and variance of σ^2
$\mathcal{U}(x, y)$	uniform distribution taking values between x and y
$\mathbf{x} \in \mathcal{Y}$	\mathbf{x} is element of the set \mathcal{X}
$\mathbf{x}^t, \mathbf{X}_t$	values of the vector \mathbf{x} and matrix \mathbf{X} at the t^{th} iteration, respectively