

**PERFORMANCE EVALUATION OF
MULTI-ANTENNA SYSTEMS: ANALYTICAL
AND MACHINE LEARNING APPROACHES**

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**BHARTI SCHOOL OF TELECOMMUNICATION
TECHNOLOGY AND MANAGEMENT
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by

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**BHARTI SCHOOL OF TELECOMMUNICATION
TECHNOLOGY AND MANAGEMENT**

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Certificate

This is to certify that the thesis entitled “**Performance Evaluation of Multi-Antenna Systems: Analytical and Machine Learning Approaches**” being submitted by **Pialy Biswas** to the Bharti School of Telecommunication Technology and Management, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of the bona-fide research work carried out by her under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.



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Abstract

The demand for a higher data rate in wireless communications has motivated the study of the usage of multiple antennas both at the transmitter and the receiver side. Multiple-input multiple-output (MIMO) systems assure higher reliability and spectral efficiency than single-input single-output systems due to their diversity and spatial multiplexing gains. With increasing system capacity, massive MIMO and multi-user MIMO (MU-MIMO) have applications in networks beyond 5G and 6G. The existing communication technologies are based on coherent detection, which needs significant amounts of signaling specifically for massive MIMO. Therefore, alternative approaches to overcome the drawbacks of coherent massive MIMO systems are noncoherent detection and differential detection. In addition to that, machine learning algorithms can be one of the effective tools for enhancing the performance of multi-antenna systems. There are three main categories of machine learning techniques: supervised learning (SL), unsupervised learning, and reinforcement learning. SL trains models using labeled data, whereas unsupervised learning infers patterns from unlabeled data.

In this thesis, we study both single-user (SU) and MU-MIMO systems under a flat Rayleigh fading environment. Here, we focus on designing an optimal precoder analytically for SU-MIMO systems with the Kronecker product correlation model using noncoherent detection and differential detection. Firstly, for the case of noncoherent reception, we optimize precoder parameters considering a spatially multiplexed SU-MIMO system with constellations $\{0, 1\}$ and $\{1, -r\}$ (with $0 \leq r < 1$). The saturation value of the symbol vector error probability at high signal-to-noise ratio (SNR) is minimized with respect to the precoder parameters and r . It is found from computation that the optimal precoder parameters are approximately in geometric progression,

which simplifies the optimization problem. Furthermore, computational results show that for a large number of transmit antennas, the optimal value of r in the constellation $\{1, -r\}$ tends to 0. Secondly, we study deterministic precoding and combining for an SU-MIMO system where we use differential encoding at the transmitter and differential detection with a maximum likelihood decision rule at the receiver. The precoder is designed to maximize the bound on the average received SNR, whereas the combiner is structured to minimize the union bound on the symbol error probability.

We next extend our work by implementing machine learning algorithms in cellular and cell-free (CF) networks to obtain performance better than that of traditional systems. Specifically, we focus on an SL based method for symbol detection in an MU single-input multiple-output (MU-SIMO) system and an unsupervised learning based access point (AP) clustering technique in a CF massive MIMO (CFMM) system. For supervised learning, an extreme learning machine (ELM) is chosen as the equalizer at the receiver end due to its super-fast learning ability and minimum training error. The network is trained online with known pilot symbols and then used to detect unknown transmit symbols using the parameters learned during training. To reduce the pilot requirement of the proposed model, we discuss an alternative method where the channel is estimated using a small number of pilot symbols and the ELM network is trained using a separate training dataset along with the channel estimate. Furthermore, we demonstrate how the proposed ELM equalizer outperforms existing linear equalizers, nonlinear detection techniques, and deep learning based models in terms of error rate and computational complexity. For the case of unsupervised learning, we use the Gaussian mixture model to cluster the APs on the basis of large-scale fading coefficients. We present an optimization problem that optimizes both the upper bound on the average downlink rate per user and the number of clusters. The computation expense is much lower than the current techniques, since the existing methods require evaluations of network performance in multiple iterations to find the optimal number of clusters. Additionally, we study the statistics of the spectral efficiency (SE) per user of clustered CFMM, and the SE per user can be approximated by the logistic distribution.

सार

वायरलेस संचार में उच्च डेटा दर की मांग ने ट्रांसमीटर और रिसीवर दोनों तरफ एकाधिक एंटेना के उपयोग के अध्ययन को प्रेरित किया है। मल्टीपल-इनपुट मल्टीपल-आउटपुट (एमआईएमओ) सिस्टम अपनी विविधता और स्थानिक मल्टीप्लेक्सिंग लाभ के कारण सिंगल-इनपुट सिंगल-आउटपुट सिस्टम की तुलना में उच्च विश्वसनीयता और वर्णक्रमीय दक्षता सुनिश्चित करते हैं। बढ़ती सिस्टम क्षमता के साथ, बड़े पैमाने पर एमआईएमओ और बहु-उपयोगकर्ता एमआईएमओ (एमयू-एमआईएमओ) के अनुप्रयोग ५जी और ६जी नेटवर्क से परे हैं। मौजूदा संचार प्रौद्योगिकियां सुसंगत पहचान पर आधारित हैं, जिन्हें विशेष रूप से बड़े पैमाने पर एमआईएमओ के लिए महत्वपूर्ण मात्रा में सिग्नलिंग की आवश्यकता होती है। इसलिए, सुसंगत विशाल एमआईएमओ सिस्टम की कमियों को दूर करने के लिए वैकल्पिक दृष्टिकोण नॉनकोहीरेंट डिटेक्शन और डिफरेंशियल डिटेक्शन हैं। इसके अलावा, मल्टी-एंटीना सिस्टम के प्रदर्शन को बढ़ाने के लिए मशीन लर्निंग एल्गोरिदम प्रभावी उपकरणों में से एक हो सकता है। मशीन लर्निंग तकनीकों की तीन मुख्य श्रेणियां हैं: सुपरवाइज्ड लर्निंग (एसएल), अनसुपरवाइज्ड लर्निंग, और रीइंफोर्समेंट लर्निंग। एसएल लेबल किए गए डेटा का उपयोग करके मॉडल को प्रशिक्षित करता है, जबकि अनसुपरवाइज्ड लर्निंग बिना लेबल वाले डेटा से पैटर्न का अनुमान लगाता है।

इस थीसिस में, हम एक फ्लैट रेले फेडिंग वातावरण के तहत एकल-उपयोगकर्ता (एसयू) और एमयू-एमआईएमओ दोनों प्रणालियों का अध्ययन करते हैं। यहां, हम नॉनकोहीरेंट डिटेक्शन और डिफरेंशियल डिटेक्शन का उपयोग करके क्रोनेकर उत्पाद सहसंबंध मॉडल के साथ एसयू-एमआईएमओ सिस्टम के लिए विश्लेषणात्मक रूप से एक इष्टतम प्रीकोडर डिजाइन करने पर ध्यान केंद्रित करते हैं। सबसे पहले, नॉनकोहीरेंट रिसेप्शन के मामले में, हम कॉन्स्टेलेशन $\{0, 1\}$ और $\{1, -r\}$ ($0 \leq r < 1$ के साथ) के साथ स्थानिक रूप से बहुसंकेतन एसयू-एमआईएमओ प्रणाली पर विचार करते हुए प्रीकोडर मापदंडों को अनुकूलित करते हैं। उच्च सिग्नल-टू-शोर अनुपात (एसएनआर) पर संकेत वेक्टर त्रुटि संभावना का संतृप्ति मूल्य प्रीकोडर पैरामीटर और r के संबंध में कम से कम किया गया है। गणना से यह पाया गया है कि इष्टतम प्रीकोडर पैरामीटर लगभग ज्यामितीय प्रगति में हैं, जो अनुकूलन समस्या को सरल बनाता है। इसके अलावा, गणनात्मक परिणाम बताते हैं कि बड़ी संख्या में ट्रांसमीट एंटेना के लिए कॉन्स्टेलेशन $\{1, -r\}$ में r का इष्टतम मान 0 हो जाता है। दूसरे में, हम एक एसयू-एमआईएमओ सिस्टम के लिए निर्धारित प्रीकोडिंग और कंबाइनिंग का अध्ययन करते हैं जहां हम ट्रांसमीटर में डिफरेंशियल एनकोडिंग और रिसीवर में एक अधिकतम संभावना निर्णय नियम के साथ डिफरेंशियल डिटेक्शन का उपयोग करते हैं। प्रीकोडर को औसत प्राप्त एसएनआर पर बाउंड को अधिकतम

करने के लिए डिज़ाइन किया गया है, जबकि कॉम्बिनेर को संकेत त्रुटि संभावना पर यूनियन बाउंड को कम करने के लिए संरचित किया गया है।

हम पारंपरिक प्रणालियों की तुलना में बेहतर प्रदर्शन प्राप्त करने के लिए सेलुलर और सेल-फ्री (सीएफ) नेटवर्क में मशीन लर्निंग एल्गोरिदम को लागू करके अपने काम का विस्तार करते हैं। विशेष रूप से, हम एमयू सिंगल-इनपुट मल्टीपल-आउटपुट (एमयू-एसआईएमओ) सिस्टम में संकेत का पता लगाने के लिए एसएल आधारित विधि और सीएफ मैसिव एमआईएमओ (सीएफएमएम) सिस्टम में एक अनसुपरवाइज्ड लर्निंग आधारित एक्सेस प्वाइंट (एपी) क्लस्टरिंग तकनीक पर ध्यान केंद्रित करते हैं। सुपरवाइज्ड लर्निंग के लिए, एक एक्सट्रीम लर्निंग मशीन (ईएलएम) को इसकी सुपर-फास्ट सीखने की क्षमता और न्यूनतम प्रशिक्षण त्रुटि के कारण रिसीवर के अंत में इक्वलाइज़र के रूप में चुना जाता है। नेटवर्क को ज्ञात पायलट संकेतों के साथ ऑनलाइन प्रशिक्षित किया जाता है और फिर प्रशिक्षण के दौरान सीखे गए मापदंडों का उपयोग करके अज्ञात ट्रांसमीट संकेतों का पता लगाने के लिए उपयोग किया जाता है। प्रस्तावित मॉडल की पायलट आवश्यकता को कम करने के लिए, हम एक वैकल्पिक विधि पर चर्चा करते हैं जहां चैनल का अनुमान कम संख्या में पायलट संकेतों का उपयोग करके लगाया जाता है, और ईएलएम नेटवर्क को चैनल अनुमान के साथ एक अलग प्रशिक्षण डेटासेट का उपयोग करके प्रशिक्षित किया जाता है। इसके अलावा, हम प्रदर्शित करते हैं कि कैसे प्रस्तावित ईएलएम इक्वलाइज़र त्रुटि दर और कम्प्यूटेशनल जटिलता के संदर्भ में मौजूदा लीनियर इक्वलाइज़र, नॉनलाइनियर डिटेक्शन तकनीक और डीप लर्निंग मॉडल से बेहतर है। अनसुपरवाइज्ड लर्निंग के मामले में, हम लार्ज-स्केल फेडिंग कोएफिसिएंट्स के आधार पर एपी को क्लस्टर करने के लिए गॉसियन मिश्रण मॉडल का उपयोग करते हैं। हम एक अनुकूलन समस्या प्रस्तुत करते हैं जो प्रति उपयोगकर्ता औसत डाउनलिनक दर की अपर बाउंड और क्लस्टर की संख्या दोनों को अनुकूलित करती है। इस पर आने वाला गणना खर्च वर्तमान तकनीकों की तुलना में बहुत कम है क्योंकि मौजूदा तकनीकों के लिए क्लस्टर की इष्टतम संख्या खोजने के लिए एकाधिक इटरेरेशन्स में नेटवर्क प्रदर्शन के मूल्यांकन की आवश्यकता होती है। इसके अतिरिक्त, हम क्लस्टर्ड सीएफएमएम के प्रति उपयोगकर्ता स्पेक्ट्रल एफिशिएंसी (एसई) के आंकड़ों का अध्ययन करते हैं, और प्रति उपयोगकर्ता एसई का अनुमान लॉजिस्टिक वितरण द्वारा प्राप्त किया जा सकता है।

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