PROPERTIES OF THRESHOLD FUNCTIONS
OF MORE THAN EIGHT VARIABLES,
AND THEIR DETECTION

by

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C E R T I F I C A T E

Certified that the thesis "Properties of Threshold Functions of More than Eight Variables, and their Detection", which is being submitted by Shri Sureshchander for the award of the degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

July 22, 1974

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A C K N O W L E D G E M E N T

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S Y N O P S I S

The gates based on threshold decision principle hold promise in the synthesis of logical systems, especially digital computers. The popularity of these gates has been rather limited because of poor signal to noise ratio, and the fact that precision components are required to implement threshold logic gates. However, with advances in technology, especially in the field of ICs, it should be possible to have reliable and inexpensive threshold gates in the near future.

The properties of switching functions which are threshold or 1-realizable have been extensively studied by various research workers for n $\leq$ 8, where n is the number of variables in a switching function. The threshold functions of nine or more variables have not been studied in any meaningful way as their properties are almost unknown. The reason why a function of nine or more variables should be tested for 3-asummability, besides 2-asummability, (for checking for 1-realizability) has not been known so far. However, this fact has been established by exhaustive enumeration and counter examples The functions of nine or more variables could not be studied because of enormous computation required to test m-asummability, strictly speaking, m, here, is unbounded,

though m is finite for finite n, i.e., there exists an m such that if a function is m-asummable, then it is also k-asummable, for $k \geq m$. But unfortunately the bound on m has not been established so far, in general.

In this thesis, properties of threshold functions have been investigated, in general. In particular, this study is devoted to properties of threshold functions of nine or more variables. The necessary and sufficient conditions for the 1-realizability of functions upto 15 variables have been obtained. The reason for restricting the study upto 15 variables is due to the fact that no mathematical relationship between m and n, in general, could be found. However, a condition has been established under which an (m - 1)-asummable function may become m-summable function. Further, this condition has been used to find the values of m for functions upto 15 variables.

It was found that 3-asummability is a necessary and sufficient condition for 1-realizability for 9 and 10 variable functions, whereas functions must be tested for 4-asummability for n = 11, 12 and 13, and for 5-asummability for n = 14 and 15.

The S_T class of functions, viz., the threshold functions, is a subset of the S_C[13] class of functions

(Definition 2.2,2), i.e., $S_T \subseteq S_C$. It is easier to test for membership in the S_C class of functions than in the S_C class of functions. It is of advantage to know the condition(s) under which both these classes are identical. It has been shown that $S_C = S_T$ for $n \leq 14$

The property of <u>superunateness</u> has been introduced to test the asummability of switching functions. Superunateness, along with other properties of threshold functions (described in this thesis), has been used to construct tables for testing 3-asummability of switching functions of 9 and 10 variables. These tables have reduced the overall computational effort, for checking 3-asummability, by a factor of $\approx 10^6$ and about 67,000 for 9 and 10 variables respectively.

A new property has been described, i.e., a <u>pivot-vertex</u> (there are only two pivot vertices for each n), defined in this thesis, should be TRUE if a pseudo-major and pseudo-canonical function is to be 1-realizable. This is a very useful property for preliminary checking of functions for 1-realizability.

Also, fast techniques have been proposed for testing 2-asummability as testing for this property is required for all n.

In the course of this investigation, certain other properties of switching functions have been, also, examined. These are of advantage in testing for 1-realizability of switching functions, also. The algorithms so developed are for testing i) unateness, ii) redundancy of variables, iii) partial and total symmetries, and iv) for minimization of switching functions. These algorithms are given in the appendices,

C O N T E N T S