TOPOLOGY OPTIMIZATION OF CONTINUUM STRUCTURES USING GENETIC ALGORITHMS

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N. SWAMINATHAN
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The results contained in this thesis have not been submitted in part or full, to any University or Institute for the award of degree or diploma.

C.V. Ramakrishnan

C.V. Ramakrishnan

Professor,

Department of Applied Mechanics

Indian Institute of Technology

New Delhi – 110 016

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(N.Swaminathan)

ABSTRACT

Design optimization has been researched extensively for over fifty years because of its importance in the Aerospace, Structural and Automotive fields. The

constraints on displacement, stresses and natural frequencies, under given load and support conditions. Recently, much research has been focused on optimum topology design since it yields maximum performance improvements. However, most of the methods available today have limited applicability and either get stuck up in local optima, or, in the other extreme, do not explore other possible equally good designs. The strong need for a simple, robust and unified approach to solve topology optimization problems in all possible frameworks – including multiple load cases and multi-objective formulations – spurred the consideration of a global search method, like Genetic Algorithms (GA), as a possible solution. The current state of the art, though, reveals several problems regarding performance of GA in the context of topology design like noisy designs, difficulty in handling constraints and high computational demand. The present investigation is inspired by the aforementioned scenario.

Mesh independence is demonstrated by refining the mesh but maintaining the perimeter limit fixed. Control of cellular design by tightening perimeter limit is demonstrated. Optimized topology is sought for various volume fractions. Alternate optima are captured wherever they exist. Proper balancing of penalty functions with objective function is achieved through normalization of the former. Effect of

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Mesh independence is demonstrated by refining the mesh but maintaining the perimeter limit fixed. Control of cellular design by tightening perimeter limit is demonstrated. Optimized topology is sought for various volume fractions. Alternate optima are captured wherever they exist. Proper balancing of penalty functions with objective function is achieved through normalization of the former. Effect of

varying the normalization factors on the designs is studied. Invalid designs are detected, quantified and penalized. Effect of penalizing invalid topologies is studied.

For problems with expected truss-like design solutions, a different solution approach using stress deviation minimization is successfully implemented. This has also resulted in control of cellular designs.

A number of new genetic operators have been made and used throughout the work resulting in improved results.

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and has rendered the scalability of GA to larger problems possible.

Problem of finding optimum topologies for multiple load cases has been formulated as a multi-objective problem and Pareto front obtained. An alternative single objective dual formulation has also been implemented and identical topologies are obtained. The multi-objective optimization approach has been shown to be an alternative to constraint handling. Both the approaches are very robust and versatile and yield correct solutions. The Pareto optimal solutions are obtained faster.

More general 3-D problems have been taken up to test and demonstrate the robustness of the design optimization tool developed.

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