

OPTIMIZATION STUDIES IN VISCOELASTICALLY
DAMPED SANDWICH BEAMS AND PLATES

Thesis submitted to the Indian Institute of Technology, Delhi,
for the award of the degree of
DOCTOR OF PHILOSOPHY

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May, 1984

ABSTRACT

The present study deals with the vibration and damping analysis of beams and plates with constrained viscoelastic layers, with the objective of carrying out optimization studies for such configurations.

The optimization studies for maximizing the modal system loss factors and minimizing the displacement response are carried out for a fully covered sandwich plate consisting of two equally thick skin faces with a constrained viscoelastic layer. The thicknesses and mass densities of skin and core layers and temperature are taken as design variables. Linear relationship between the material mass density and Young's modulus is assumed. The effect of temperature and frequency on the core properties are taken into account. The constraints on the design variables and parameters such as mass per unit area, static stiffness and resonant frequency are considered. The optimization of the core material for a viscoelastically damped sandwich plate is also carried out for the maximum modal system loss factor. The dynamic characterization of the viscoelastic material for harmonic motion, represented by the complex shear modulus is taken in terms of a model comprising of springs and dashpots. The parameters of a four element model subjected to constraints for maximum modal system loss factors have been determined.

The investigations for the damping effectiveness of partially covered beams with constrained viscoelastic layer are carried out by Markus method and Rayleigh-Ritz method. In these

methods, approximate mode shapes of the beams are assumed. The former method gives the modal system loss factor, while, the latter method results in a complex eigenvalue problem giving the resonant frequencies and the associated modal system loss factors. Classical-cum-search method for a beam with single patch has been developed for determination of resonant frequency and modal system loss factor for the exact classical solution. Assumption for mode shape is not required in this case. The results obtained from the three methods are compared. Studies for optimizing the location, geometric and material properties, and the percentage of the constrained viscoelastic coverage have been carried out. The effect of providing the cuts in the coverage of a fully covered beam is also investigated.

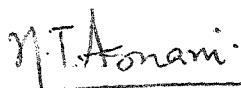
Rayleigh-Ritz's approach is extended for the analysis of a partially covered simply supported rectangular sandwich plate and the resonant frequencies and the associated modal system loss factors for the first four modes are evaluated. The studies for optimizing the location, the geometric and the material properties, and the percentage of the constrained viscoelastic coverage have been reported.

Finally, the general discussions and conclusions of the present work and the scope for the further work are given.

CERTIFICATE

This is to certify that the Thesis entitled "Optimization Studies in Viscoelastically Damped Sandwich Beams and Plates" being submitted by Mr. Arun K. Lall to the Indian Institute of Technology, Delhi, for the award of the Degree of 'Doctor of Philosophy' in Mechanical Engineering, is a record of bonafide research work carried out by him. He has worked under our guidance and supervision and has fulfilled the requirements for the submission of this Thesis, which has reached the requisite standard.

The results contained in this Thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



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ACKNOWLEDGEMENTS

The author wishes to express his deep sense of gratitude to Prof. B.C. Nakra and Prof. N.T. Asnani of Mechanical Engineering Department for providing valuable, inspiring and constructive guidance at all the stages of this study. Their continuous encouragement made the task lighter.

Thanks are due to the staff of the Computer Centre for their cooperation in the computation work.

Finally, the author offers an inadequate acknowledgement of his appreciation, to his colleagues, who very willingly assisted in some way or the other in the completion of this Thesis.

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