COMPUTER ASSISTED GENERATION OF EXACT STIFFNESS MATRIX AND FIXED EDGE FORCES FOR CERTAIN AXISYMMETRIC PLATE AND SHELL ELEMENTS

by

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CERTIFICATE

This is to certify that the thesis entitled "Computer Assisted Generation of Exact Stiffness Matrix and Fixed Edge Forces for Certain Axisymmetric Plate and Shell Elements", being submitted by Mr. Rajendrapal Singh Bhatia to the Indian Institute Of Technology, Delhi, for the award of Degree of Doctor of Philosophy, is a record of bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for the thesis, which to my knowledge, has reached the requisite standard.

The results contained in this thesis have not been submitted in part or full, to any other university or institute for the award of any Degree or Diploma.

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PREFACE

Axisymmetric thin shell structures are commonly used in engineering practice. Frequently such structures are composed of axisymmetric segments such as annular plates, cylindrical shells, conical and spherical shell frustums. The finite element method is quite suited to the task of carrying out elastic analysis of these composite axisymmetric shell structures. An essential step in the finite element method is the determination of elemental stiffness matrix. Conventionally this is done by modelling a given composite axisymmetric shell structure in terms of a large number of closely spaced conical shell frustums. Expressions for stiffness coefficients are usually found by assuming approximate displacement fields for the element. However this procedure is rather wasteful of the computer time and data-preparation effort owing to the necessity of using a fine mesh. An alternative approach is the use of so-called "macro elements", which can model the whole structure by means of much fewer elements. In fact each element may be as large as a complete segment of the shell structure. However this approach places considerably more stringent demands on accuracy of the stiffness matrix. Although analytical studies on the elastic behaviour of some of the commonly used shell segments are available in the published literature, they can not be directly used for deriving elemental stiffness matrix since derivation of stiffness coefficients from analytical solutions is, in general, a complicated and cumbersome task.

The present work is concerned with the development of a procedure of generating exact stiffness matrix and fixed edge forces for axisymmetric shell elements. Explicit expressions for stiffness coefficients are not sought because they tend to be too lengthy and too intricate. Rather the aim is to evaluate them through an algorithmic approach implementable on the computer itself. The procedure starts with an analytical solution of the differential equations governing the elastic behaviour of the shell element. Matrix equations for displacement and stress resultant are worked out from the analytical solution. These equations are in terms of (a) two intermediate square matrices called [A] and [B], and (b) two vectors representing particular integral terms of the elastic solution. Matrices [A] and [B] can be evaluated with the help of geometrical dimensions and material properties of the given shell segment. When computing the different coefficients of vectors representing particular integrals, distributed load data is also used. The matrix product $[B][A]^{-1}$ is the required stiffness matrix and is computed numerically. The fixed edge forces are evaluated for the element under given loading with the help of stiffness matrix and particular integrals for displacements and stresses.

The proposed method has been applied to a variety of elements namely a uniformly thin axisymmetric annular plate, a cylinder and a conical shell frustum. Frustum of spherical shell has also been investigated using the method but by starting from an approximate analytical solution since the exact solution was rather inconvenient to use. Validity checks on all elements have been performed. Four real-life axi-symmetric composite shell structures have also been analyzed with the help of a finite element package incorporating the proposed procedure. Wherever possible, the predicted results have been compared with those available in literature. It is found that the method is accurate and efficient.

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